4: Long-Run Relationships (ARDL)

GECO 6281 Advanced Econometrics 1 (Lab)

Patrick Mokre

Fall 2019

Auto-Regressive Distributed Lag Models

- Model the relationship between variables in a single-equation setup
- Error Correction Representation is equivalent to co-integration of non-stationary variables
- EC representation is used to test for a long-run cointegrating relationship
- This allows for testing without knowing if the co-integrating variables are I(0) or I(1) themselves
- Examples: Wages and Labor Productivity, Foreign Direct Investment and Capital Intensity

Engle-Granger (1987) Test for long-run relationships

Assume $(y_t, x_t)'$ is a vector of I(1) variables

First Step: Run levels OLS $y_t = \alpha_1 + x_t'\beta + v_t$

Test if v_t is stationary (e.g. Adjusted Dickey Fuller or KPSS test)

Second Step: Estimate an error correction model and include lagged residuals $\hat{v}_{t-1}$ (if they are stationary):

$$\Delta y_t = \alpha_2 + \gamma \hat{v}_{t-1} + \sum_{i}^{p-1} \phi_{yi} \Delta y_{t-i} + \sum_{j}^{p-1} \phi_{xj} \Delta x_{t-j} + u_t$$

Test whether $-1 \leq \gamma < 0$.

Engle-Granger (1987): Downsides

variables must be I(1) and tested beforehand.

In short panels, first-step OLS estimates may be biased because of omitted short-run dynamics (no x_t as covariate), which influences the second step.

Standard significance testing in the first step is not available because asymptotic distribution of $\hat{\beta}$ is non-normal.

Engle-Granger (1987): Application

```
use usa.dta, clear
gen date = tq(1984q1) + _n-1
tsset date

dfuller f
dfuller D.f     //  f is integrated of order 1

dfuller b
dfuller D.b     //  b is integrated with order 1

reg b f
predict e, resid
dfuller e, noconstant
tsline e
```

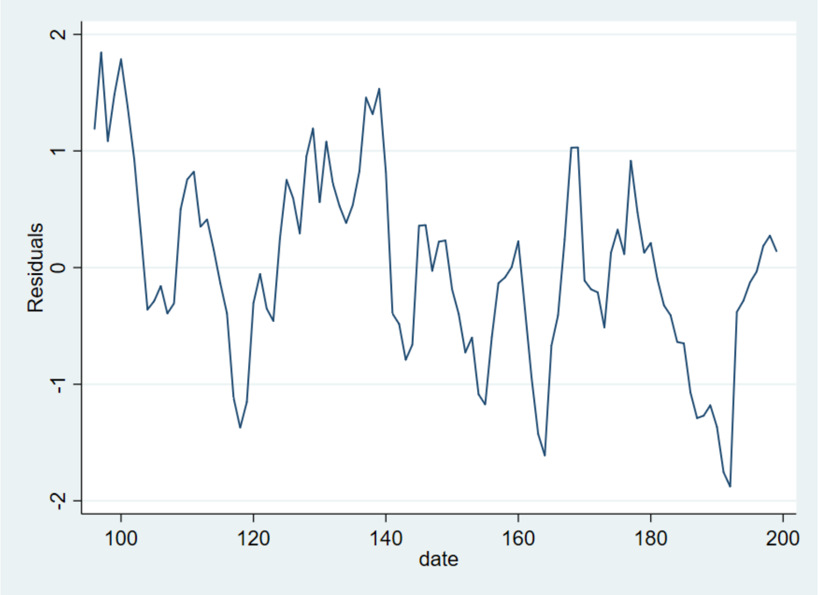
Engle-Granger (1987): Results

```
. dfuller e, noconstant

Dickey-Fuller test for unit root                   Number of obs   =       103

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -3.188            -2.600            -1.950            -1.610
```

Engle-Granger (1987): Results 2



Possible Models for long-run relationships

(https://davegiles.blogspot.com/2013/06/ardl-models-part-ii-bounds-tests.html)

If one wants to understand the dynamic relationship between two variables, there is a number of possible cases:

- Both are I(0), i.e. stationary. Then an OLS on the variable levels will be unbiased and efficient.
- The variables are integrated of the same order (eg. I(1)) but *not cointegrated*. Appropriate differentiation (i.e. first difference for first order integration) allows for OLS estimation.
- The variables are integrated of the same order *and* co-integrated. Then a level OLS provides the long-run relationship, whereas an Error Correction Model (ECM) (which can be estimated using OLS) represents the short-run dynamics.
- Data might be of different orders and/or co-integrated ("things are not as clear cut"). ARDL analyzes *both* short-run dynamics and long-run relationships.

ARDL: Pre-Requisites and Procedure

- none of the variables must be I(2)
- The model is written as an unrestricted ECM
  $\Delta y_t = \alpha + \sum_i^{p-1} \beta_1 \Delta y_{t-i} + \sum_j^{p-1} \beta_2 \Delta x_{t-j} + \gamma_1 y_{t-1} + \gamma_2 x_{t-1} + \epsilon_t$
- an appropriate lag structure is determined, e.g. using information criteria
- test for *serially independent errors*
- test for *dynamic stability*
- Pesaran-Shin-Smith Bounds test for long-run relationship (later in semester)
- estimate long-run "levels" model and short-run ECM

ARDL in STATA

```
. ardl eur us, aic //Use Akaike Information Criterion to decide on optimal mode

ARDL(4,0) regression

Sample:       424 -       614                     Number of obs     =        191
                                                  F(   5,     185)  =    3004.38
                                                  Prob > F          =     0.0000
                                                  R-squared         =     0.9878
                                                  Adj R-squared     =     0.9875
Log likelihood = -75.256023                       Root MSE          =     0.3646

------------------------------------------------------------------------------
         eur |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         eur |
         L1. |   1.055477   .0698777    15.10   0.000     .9176176    1.193337
         L2. |   .0733688   .1042555     0.70   0.482    -.1323138    .2790513
         L3. |   .1138271   .1057457     1.08   0.283    -.0947953    .3224496
         L4. |  -.2835677   .0690439    -4.11   0.000    -.4197823    -.147353
             |
          us |   .0526194   .0142578     3.69   0.000     .0244906    .0807482
       _cons |  -.0077393   .0609519    -0.13   0.899    -.1279894    .1125109
------------------------------------------------------------------------------
```

ARDL in STATA 2: Interpretation

(http://repec.org/usug2018/uk18_Kripfganz.pdf)

$$y_t = \alpha_0 + \alpha_1 t + \sum_i^p \phi_i y_{t-i} + \sum_j^q \beta_j' x_{t-j} + u_t$$

- Coefficients represent the long-term relationship between variable levels
- Include auto-regressive terms
- Include a time trend (trend stationarity)

ARDL in STATA 3 (ECM)

```
. ardl eur us, bic ec

ARDL(4,0) regression

Sample:        424 -       614                   Number of obs     =        191
                                                 R-squared         =     0.2943
                                                 Adj R-squared     =     0.2753
Log likelihood = -75.256023                      Root MSE          =     0.3646

------------------------------------------------------------------------------
       D.eur |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ADJ          |
         eur |
         L1. |  -.0408945   .0103098    -3.97   0.000    -.0612345   -.0205546
-------------+----------------------------------------------------------------
LR           |
          us |   1.286711   .3021187     4.26   0.000     .6906697    1.882751
-------------+----------------------------------------------------------------
SR           |
         eur |
         LD. |   .0963718   .0681707     1.41   0.159    -.0381202    .2308637
        L2D. |   .1697405   .0678472     2.50   0.013     .0358869    .3035941
        L3D. |   .2835677   .0690439     4.11   0.000      .147353    .4197823
             |
       _cons |  -.0077393   .0609519    -0.13   0.899    -.1279894    .1125109
```

ARDL in STATA 4: Interpretation of the Conditional EC Formulation

$$\Delta y_t = \alpha_0 + \alpha_1 t + \alpha_2(y_{t-1} - \theta x_t) + \sum_{i=1}^{p-1} \psi_{yi} \Delta y_{t-i} + \sum_{i=0}^{q-1} \psi'_{xi} \Delta x_{t-i} + u_t$$

- α_2 is the *speed-of-adjustment* parameter, measuring how fast the system returns to equilibrium. It is denoted as a negative ("ADJ").
- $\alpha_2 = 1 - \sum_{j=1}^{p} \phi_j$ (from the level-ARDL regression).
- $\theta = \frac{\sum_{j=0}^{q} \beta_j}{\alpha_2}$ denotes the long run coefficients from the same *first step*. ("LR")
- ψ just denote the short-run coefficients from the second, error-correcting step ("SR")

ARDL in STATA 5: Alternative Error Correction Representation

```
. ardl eur us, bic ec1

ARDL(4,0) regression

Sample:       424 -      614                     Number of obs     =        191
                                                 R-squared         =     0.2943
                                                 Adj R-squared     =     0.2753
Log likelihood = -75.256023                      Root MSE          =     0.3646

------------------------------------------------------------------------------
       D.eur |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ADJ          |
         eur |
         L1. |  -.0408945   .0103098    -3.97   0.000    -.0612345   -.0205546
-------------+----------------------------------------------------------------
LR           |
          us |
         L1. |   1.286711   .3021187     4.26   0.000     .6906697    1.882751
```

ARDL in STATA 6: Alternative Error Correction Representation

```
-------------+----------------------------------------------------------------
SR           |
         eur |
         LD. |   .0963718   .0681707     1.41   0.159    -.0381202    .2308637
        L2D. |   .1697405   .0678472     2.50   0.013     .0358869    .3035941
        L3D. |   .2835677   .0690439     4.11   0.000      .147353    .4197823
             |
          us |
         D1. |   .0526194   .0142578     3.69   0.000     .0244906    .0807482
             |
       _cons |  -.0077393   .0609519    -0.13   0.899    -.1279894    .1125109
------------------------------------------------------------------------------
```

ARDL in STATA 7: Alternative Error Correction Representation

$$\Delta y_t = \alpha_0 + \alpha_1 t + \alpha_2 (y_{t-1} - \theta x_{t-1}) + \sum_{i=1}^{p-1} \psi_{yi} \Delta y_{t-i} + \omega' \Delta x_{t-1} + \sum_{i=1}^{q-1} \psi'_{xi} \Delta x_{t-i} + u_t$$

- Δx_{t-1} is isolated with coefficient ω (“SR”: “D1”)
- Thus, the long-run dynamics only include lag levels (“LR”: “L1”).

Extending Arellano-Bond 1 (Repetition)

A dynamic panel model can be written in fixed effects.

$$y_{it} = \alpha_i + \sum_{j}^{p} \gamma_i y_{it-j} + x_t'\beta + \epsilon_{it}$$

y can be correlated (1) directly thorugh lags of y ("true state dependency"), (2) directly thorugh x ("observed heterogeneity") or (3) indirectly through individual effects α_i ("unobserved heterogeneity"). Keep in mind that individual effects respond to unobserved characteristics.

Note that mean difference ("within") is inconsistent, as is instrumented mean difference estimation, as mean differences will be correlated with the mean error term.

First Difference estimation is also inconsistent, but instrumented fist difference estimation is permitted.

$$\Delta y_{it} = \sum_{j}^{p-1} \gamma_j \Delta y_{i,t-j} + \Delta x_t'\beta + \Delta\epsilon_{it}$$

Note that $\Delta\epsilon_{it} = \epsilon_{i,t} - \epsilon_{it-1}$ is correlated with $\Delta y_{it-1} = y_{it-1} - y_{it-2}$.

Extending Arellano-Bond 2 (Repetition)

- Anderson-Hsiao: y_{t-2} is uncorrelated with $\Delta\epsilon_{it}$ and can be used as an instrument for Δy_{it-1}
- Arellano-Bond: Adding more lags as instruments makes estimation more efficient
- Using the General Method of Moments (GMM) is even more efficient. Restricting lags in long and narrow samples (large T) increases asymptotic performance. `vce(robust)` includes Windmeijer (2005) robust standard errors.

Extending Arellano-Bond 3 (Repetition 2)

```
. xtabond lwage, lags(2) twostep vce(robust)

Arellano-Bond dynamic panel-data estimation     Number of obs     =      2,380
Group variable: id                              Number of groups  =        595
Time variable: t
                                                Obs per group:
                                                              min =          4
                                                              avg =          4
                                                              max =          4

Number of instruments =     15                  Wald chi2(2)      =    1974.40
                                                Prob > chi2       =     0.0000
Two-step results
                                      (Std. Err. adjusted for clustering on id)
------------------------------------------------------------------------------
             |              WC-Robust
       lwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lwage |
         L1. |   .6095931   .0330542    18.44   0.000      .544808    .6743782
         L2. |   .2708335   .0279226     9.70   0.000     .2161061    .3255608
             |
       _cons |   .9182262   .1339978     6.85   0.000     .6555952    1.180857
------------------------------------------------------------------------------
Instruments for differenced equation
        GMM-type: L(2/.).lwage
```

Extending Arellano-Bond 3

Both Arellano-Bover and Blundell-Bond introduce a restriction $E(\Delta y_{it-1}\epsilon_{it}) = 0$ such that Δy_{it-1} can be introduced as an instrument.

This is a solution for the problem that the pure Arellano-Bond instruments tend to suffer from weak instrumental variable problems.

```
use mus08psidextract.dta, clear
 xtdpdsys lwage, lags(2) twostep

System dynamic panel-data estimation          Number of obs      =      2,975
Group variable: id                            Number of groups   =        595
Time variable: t
                                              Obs per group:
                                                           min =          5
                                                           avg =          5
                                                           max =          5

Number of instruments =     20                Wald chi2(2)       =    4174.06
                                              Prob > chi2        =     0.0000
Two-step results
------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lwage |
         L1. |   .6017105    .019114    31.48   0.000     .5642477    .6391732
         L2. |   .2880127   .0179783    16.02   0.000     .2527759    .3232496
             |
```

Compare Arellano-Bond and Arellano-Bover

```
. quietly xtabond lwage, lags(2) twostep

. estimates store abond2

. quietly xtdpdsys lwage, lags(2) twostep

. estimates store abover1

. esttab abond2 abover1, mtitles("Arellan-Bond" "Arellano-Bover")

--------------------------------------------
                    Ar.nd           Ar.er   
                    lwage           lwage   
--------------------------------------------
L.lwage             0.610***        0.602***
                  (26.70)         (31.48)   

L2.lwage            0.271***        0.288***
                  (14.30)         (16.02)   

_cons               0.918***        0.856***
                   (7.17)          (9.34)   
--------------------------------------------
N                    2380            2975   
--------------------------------------------
```

Serial Corellation

Both Arellano-Bond and Arellano-Bover/Blundell-Bond methdoologies require the error terms to be serially uncorrelated.

Autocorrelation in ϵ_{it} and ϵ_{it-1} (absent individual effects) would render y_{t-2} be endogenous to v_{it-1}.

This can be tested using `estat abond`.

```
. estat abond //Test for serial correlation of error terms

Arellano-Bond test for zero autocorrelation in first-differenced errors
  +-----------------------+
  |Order |  z     Prob > z|
  |------+----------------|
  |   1  |-4.3902  0.0000 |
  |   2  |-2.1733  0.0298 |
  +-----------------------+
   H0: no autocorrelation
```

Treating Serial Correlation in the error term

- Include more and earlier lags, the re-do the test
- Model a moving average process in the error term: $v_{it} = \epsilon_{it} + \theta v_{it-1}$
- In STATA, `xtdpd` allows for this (`dpd` denotes "dynamic panel data")

Arellano-Bover in xtdpd

Reproduce earlier model

```
xtdpd L(0/2).lwage, dgmmiv(lwage) twostep

Dynamic panel-data estimation                Number of obs      =      2,975
Group variable: id                           Number of groups   =        595
Time variable: t
                                             Obs per group:
                                                          min =          5
                                                          avg =          5
                                                          max =          5

Number of instruments =     15               Wald chi2(2)       =    1471.72
                                             Prob > chi2        =     0.0000
One-step results
------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lwage |
         L1. |   .5707517    .024875    22.94   0.000     .5219976    .6195058
         L2. |   .2675649   .0203552    13.14   0.000     .2276694    .3074605
             |
       _cons |   1.203588   .1455457     8.27   0.000     .9183232    1.488852
------------------------------------------------------------------------------
```

Arellano-Bover with Moving Average: Results 2

```
. esttab abond1 abover1 xtdpd1, mtitles("Arellano-Bond" "Arellano-Bover 1" "Are

------------------------------------------------------------
                      (1)             (2)             (3)   
             Arellano-B~d    Arellano-B~1    Arellano-B~2   
------------------------------------------------------------
L.lwage             0.946***        0.602***        0.610***
                  (82.32)         (31.48)         (26.70)   

L2.lwage                            0.288***        0.271***
                                  (16.02)         (14.30)   

_cons               0.451***        0.856***        0.918***
                   (5.93)          (9.34)          (7.17)   
------------------------------------------------------------
N                    2975            2975            2975   
------------------------------------------------------------
t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001
```

Arellano-Bover with Moving Average: Results 3

```
. estat abond //test for serial autocorrelation again

Arellano-Bond test for zero autocorrelation in first-differenced errors
  +-----------------------+
  |Order |  z     Prob > z|
  |------+----------------|
  |   1  |-4.5381  0.0000 |
  |   2  |-1.9946  0.0461 |
  +-----------------------+
   H0: no autocorrelation
```

⇒ Problem is not solved by reroducing model in a different package! (Shocker.)

Arellano-Bover with Moving Average: Results 4

```
. xtdpd L(0/2).lwage, dgmmiv(lwage, lagrange(3 4)) lgmmiv(L.lwage) twostep //ch

Dynamic panel-data estimation                    Number of obs     =     2,975
Group variable: id                               Number of groups  =       595
Time variable: t
                                                 Obs per group:
                                                              min =         5
                                                              avg =         5
                                                              max =         5

Number of instruments =     12                   Wald chi2(2)      =   4078.49
                                                 Prob > chi2       =    0.0000
Two-step results
------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lwage |
         L1. |   .7553581   .0632078    11.95   0.000      .631473    .8792432
         L2. |   .1270523   .0527304     2.41   0.016     .0237026    .2304021
             |
       _cons |   .8918238   .1147999     7.77   0.000     .6668201    1.116827
------------------------------------------------------------------------------
```

Arellano-Bover with Moving Average: Results 5

```
. esttab abond1 abover1 xtdpd1 xtdpd2, mtitles("Arellano-Bond" "Arellano-Bover

----------------------------------------------------------------------------
                      (1)             (2)             (3)             (4)   
             Arellano-B~d    Arellano-B~1    Arellano-B~2    Arellano--~1)  
----------------------------------------------------------------------------
L.lwage             0.946***        0.602***        0.605***        0.755***
                  (82.32)         (31.48)         (30.20)         (11.95)   

L2.lwage                            0.288***        0.276***        0.127*  
                                  (16.02)         (14.40)          (2.41)   

_cons               0.451***        0.856***        0.917***        0.892***
                   (5.93)          (9.34)          (9.12)          (7.77)   
----------------------------------------------------------------------------
N                    2975            2975            2975            2975   
----------------------------------------------------------------------------
t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001
```

Arellano-Bover with Moving Average: Results 6

```
. estat abond

Arellano-Bond test for zero autocorrelation in first-differenced errors
  +-----------------------+
  |Order |  z     Prob > z|
  |------+----------------|
  |   1  |-3.0078  0.0026 |
  |   2  | .22473  0.8222 |
  +-----------------------+
   H0: no autocorrelation
```

Arellano-Bover with Moving Average: Full Model

Results up to now have been shoddy at best, as we only used one variable for simplicity.

```
quietly xtabond lwage occ south smsa ind, lags(2) pre(wks, lag(1,2)) endogenous
estimates store abond_full
quietly xtdpdsys lwage occ south smsa ind, lags(2) pre(wks, lag(1,2)) endogenou
estimates store abover_full
quietly xtdpd L(0/2).lwage L(0/1).wks occ south smsa ind ms union, div(occ sout
estimates store abover_ma
```

Arellano-Bover with Moving Average: Full Model 2

```
. esttab abond_full abover_full abover_ma

------------------------------------------------------------
                      (1)             (2)             (3)
                    lwage           lwage           lwage
------------------------------------------------------------
L.lwage             0.597***        0.599***        0.851***
                  (15.98)         (21.18)          (8.95)

L2.lwage            0.250***        0.287***       0.0497
                   (8.04)          (9.94)          (0.59)

wks               -0.0155*       -0.00396        -0.00114
                  (-2.03)         (-0.65)         (-0.24)

L.wks             0.00384         0.00113        0.000108
                   (1.44)          (0.64)          (0.08)
```

Arellano-Bover with Moving Average: Full Model 3

ms	0.136	0.0347	0.0405
	(1.09)	(0.60)	(0.84)
union	-0.178	-0.0642	-0.0257
	(-1.00)	(-0.86)	(-0.37)
occ	-0.0355	-0.0458	-0.0496
	(-0.99)	(-1.39)	(-1.58)
south	-0.0101	-0.106	-0.149*
	(-0.05)	(-1.14)	(-2.00)
smsa	-0.0827	-0.0546	-0.0647
	(-1.55)	(-1.16)	(-1.23)
ind	0.0252	0.0146	0.0137
	(0.56)	(0.47)	(0.39)
_cons	1.710***	1.097**	0.890**
	(3.56)	(2.79)	(2.58)
N	2380	2975	2975

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001