Lab 4: Dynamic Panels

Econometrics Beyond Ordinary Least Squares

Patrick Mokre

WS 2020/2021

Dynamic Panel Data

- **Recap** Pooled OLS (Between Estimator), Fixed Effects (Within Estimator) and Random Effects (Weighted Mean of B and W) are **appropriate for static panel models** - no serial correlation!.
- Time Dimension is relevant for economic analysis.
- Meaningful dynamic relationships **usually introduce error autocorrelation**, ie. non-zero off-diagonal entries in the error covariance matrix.
- Including auto-regressive terms usually introduces **serial correlation**.

$$\Sigma = \begin{bmatrix} \sigma^2 & cov_1 & cov_2 \\ cov_3 & \sigma^2 & cov_4 \\ cov_5 & cov_6 & \sigma^2 \end{bmatrix}$$

Dynamic Panel Data: Example Applications

Simple **Autoregressive** term of first order.

$$y_{it} = \alpha_i + \theta y_{it-1} + \beta x_{it} + \epsilon_{it} \tag{1}$$

- Longitudinal income studies.
- Firm sales.
- GDP growth rates.

Note: No **lagged covariates** X_{it-1} in this functional form.

Dynamic Panel Estimation Problems

Estimate the model using mean differencing or first differencing $\Rightarrow$ **serially correlated error terms**.

$$(y_{it} - y_{it-1}) = (\alpha_i - \alpha_i) + \theta(y_{it-1} - y_{it-2}) + \beta(x_{it} - x_{it-1}) + (\epsilon_{it} \quad (2)$$

$$E[\epsilon_{it}(y_{it} - y_{it-1})] \neq 0; E[\epsilon_{it-1}(y_{it-1} - y_{it-2})] \neq 0; E[y_{it} y_{it-1}] \neq 0$$

$\Rightarrow E[\epsilon_{it} y_{it-1}] \neq 0 \Rightarrow E[\epsilon_{it} X_{it}] \neq 0$

Violation of exogeneity $\Rightarrow$ inconsistent OLS estimation.

Example: Above-Average impact of financialization on job losses in period T -> negative relation ship between job losses in t and $t+1$ -> job losses lower than financialization would suggest -> negative correlation between ϵ_t and ϵ_{t+1}.

Dynamic Panels: Inconsistent FE estimator

$$
\begin{aligned}
y_{it} &= \alpha_i + \gamma y_{it-1} + \epsilon_{it} \\
\hat{\gamma}_{FE} &= \frac{\sum_i^N \sum_t^T (y_{it} - \bar{y}_i)(y_{it-1} - \bar{y}_{i,-1})}{\sum_i^N \sum_t^T (y_{it-1} - \bar{y}_{i,-1})^2} \\
\bar{y}_i &= 1/N \sum_t^T y_{it} \ ; \ \bar{y}_{i,-1} = 1/N \sum_t^T y_{it-1} \\
\hat{\gamma}_{FE} &= \gamma + \frac{1/(NT) \sum_i^N \sum_t^T (\epsilon_{it} - \bar{\epsilon}_i)(y_{it-1} - \bar{y}_{i,-1})}{1/(NT) \sum_i^N \sum_t^T (y_{it-1} - \bar{y}_{i,-1})^2} \qquad (3)
\end{aligned}
$$

The **bias is positive** if if the covariance of $(\epsilon_{it} - \bar{\epsilon}_i)$ and $(y_{it-1} - \bar{y}_{it-1})$ is positive, and vice versa.

Dynamic Panels: Remedies for Serial Correlation

3 channels of inter-temporal correlation in y_i: **true state dependence** (directly $y_{i,t-1} \rightarrow y_{i,t}$), **observed heterogeneity** (directly through covariates $x_{i,t-1} \rightarrow x_{i,t} \rightarrow y_{i,t}$, or **unobserved heterogeneity** indirectly through α_i.

- **Forget about Mean Differencing**, use first differencing for FE and generalized least squares for RE.
- "Traditional" treatment for endogeneity problems: **Instrumental Variables (IV)**. For this, find instrumental variable z_i which must be **relevant** $E[x_i z_i] \neq 0$ and **independent** $E[\epsilon_i z_i] = 0$.
- Example: Unobserved characteristics in ϵ_i also affect covariates X_i, ie. firms' management style improving labor productivity.

$$y_i = \alpha + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i$$

$$E[x_{i2}\epsilon_i] \neq 0 \tag{4}$$

$$x_{i2} = \gamma + \delta z_i + \nu_i \ , \ \delta \neq 0 \tag{5}$$

$$\hat{\beta}_{IV} = \left(\sum_i^N x_i z_i \right)^{-1} \left(\sum_i^N z_i y_i \right) \tag{6}$$

Anderson-Hsiao: Second Lag Level as Instrument

Anderson and Hsiao (1981) propose 2-step least squares (2SLS) estimation of the FD estimator with $y_{i,t-2}$ as the instrument for $\Delta y_{it-1} = y_{it-1} - y_{it-2}$.

The estimator is consistent under **no serial correlation**.

$$\hat{\gamma}_{AH} = \frac{\sum_i^N \sum_t^T y_{it-2}(y_{it} - y_{it-1})}{\sum_i^N \sum_t^T y_{it-2}(y_{it-1} - y_{it-2})} \quad (7)$$

Alternatively, one can use $(y_{it-2} - y_{it-3})$ as a consistent estimator, but will **lose one period of observations**.

$$\hat{\gamma}_{AH} = \frac{\sum_i^N \sum_t^T (y_{it-2} - y_{it-3})(y_{it} - y_{it-1})}{\sum_i^N \sum_t^T (y_{it-2} - y_{it-3})(y_{it-1} - y_{it-2})} \quad (8)$$

Arellano-Bond: More Instruments!

Arellano and Bond (1991) show that more instruments **improve efficiency while retaining consistency**.

However, the model must be estimated using **panel generalized method of moments PGMM** because Δy_{t-1} is over-identified.

$$(y_{it} - y_{it-1}) = \gamma(y_{it-1} - y_{it-2}) + (x_{it} - x_{it-1})'\beta + (\epsilon_{it} -$$

$$\hat{\beta}_{AB} = \left[ ] \left(\sum_i^N \tilde{X}_i' Z_i \right)' W_N \left(\sum_i^N Z_i' \tilde{X}_i \right) \right]^{-1}$$

$$\left(\sum_i^N \tilde{X}_i' Z_i \right)' W_N \left(\sum_i^N Z_i' \tilde{y}_i \right) \quad (9)$$

$\tilde{y}_i = (y_{it} - y_{it-1})$, $\tilde{X}_i = (X_{it} - X_{it-1})$ a $(T - 2 \times K + 1)$ covariate matrix, Z_i a $(T - 2 \times r)$ instrument matrix for $\tilde{y}_i$.

W_N is a weighting matrix which is algorithmically retrieved from the **moment conditions**.