10: Co-Integration in Panels

GECO 6281 Advanced Econometrics 1 (Lab)

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Recapitulation

- Panel Data: LSDV, Fixed Effects and Random Effects
- Dynamic Panel Data & Instrumentalization: Anderson-Hsiao, Arellano-Bond, Arellano-Bover
- Auto-Regressive and Moving Average Processes
- Stationarity in Time Series and Panels
- Vector Error Correction (VECM) and Auto-Regressive Distirbuted Lag (ARDL)

Cointegration

Two time series x_t and y_t are cointegrated if they are both I(1), but a **linear combination** $\lambda_1 x_t + \lambda_2 y_t$ of the processes is I(0) stationary.

Simple Test: Regress y_t on x_t, estimate residuals $\hat{e}_t$. Then perform an augmented Dickey-Fuller test on $\hat{e}_t$ by regressing its first difference on the lagged level. The **ADF test statistic** then corresponds to the t-statistic on the corresponding coefficient.

$$y_t = \alpha + x_t'\beta + \epsilon_t \tag{1}$$

$$\Delta\hat{\epsilon}_t = a + \rho\hat{\epsilon}_{t-1} + u_t \tag{2}$$

$$ADF\epsilon_t = \frac{\hat{\rho}}{s.e.(\hat{\rho})} \tag{3}$$

Cointegration is a test on non-stationary time series to establish (or reject) a "long-run" relationship between the two.

First difference time series tend to "wander". Co-Integration implies that series are wandering together in long-run equlibrium (although groups of them can wander arbitrarily).

Johansen Unit Root Test for Cointegration 1

- Pure time series application, appropriate for vector error correction (VEC).
- i.e. not a panel cointegration test, but links time series and panel cointegration testing.
- Solves the problem of **how many co-integrating relationships** to include in the VEC.
- Let r be the number of co-integrating, n the number of all processes. $H0 : r < n$, $H_A : r = n$.
- Sequential testing for $r = (1, 2, N)$, the first non-rejection indicates the number of co-integrating processes.
- Method: Likelihood-Ratio test.

Logic (one variable vector):

$$y_t = \phi y_{t-1} + e_t \mid -y_{t-1}$$
$$\Delta y_t = (\phi - 1) y_{t-1} + e_t$$

For the unit root case $\phi = 1 \Rightarrow (\phi - 1) = 0$.

Johansen Unit Root Test for Cointegration 2

Let $y_t = (y_{1,t}, y_{2,t}, ..., y_{N,t})$ be a vector.

$$\begin{aligned} y_t &= \Phi y_{t-1} + e_t \\ \Delta y_t &= \Phi y_{t-1} - y_{t-1} + e_t \\ &= (\Phi - I) y_{t-1} + e_t \end{aligned}$$

The **rank of** $(\Phi - I)$ is equal to the number of co-integrating, non unit-root, processes: **Johansen Rank Test**. The rank of a matrix is the maximum of linearly independent column or row of a matrix (definitions are equivalent).

Both a constant term $A_0 = (a_{1,0}, a_{2,1}, ..., a_{N,0})$ and a drift term Σ can be included. Let D_t be a matrix of yearly dummy variables. Let p be some appropriate lag length.

Formulation as **long-run vector error correction model**:

$$\Delta Y_t = A_0 + \Sigma D_t + \Pi A_{t-p} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + e_t \text{ where } \Gamma_i = \sum_{j=1}^{i} [\Pi_j] - I, i = 1, ..., p-1$$

Formulation as a **transitory VECM**:

$$\Delta Y_t = A_0 + \Sigma D_t + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \Pi Y_{t-1} \text{ where } \Gamma_i = (\Pi_{i+1} + ... + \Pi_p), i = 1, ..., p-1$$

Johansen Unit Root Test for Cointegration 3

The **long-run** and the **transitory** VECM are equivalent. In both cases, testing revolves around $\Pi = \Pi_1 + ... + \Pi_p - I$.

The test finds the correct rank of Π, where $rank(\Pi) = 0$ indicates a regular first difference VAR relationship, and $rank(\Pi) = n$ indicates a VECM where all vectors are mutually cointegrated.

The STATA command for Johansen testing is `vecrank`.

Co-Integration in Panels 1

Co-Integration in panels refers to a **long-run relationship** between different variables that holds **over cross-sectional units**.

Co-Integration in Panels is to be estimated when **non-stationarity** cannot be rejected. For stationary series, there is better estimation techniques (VAR, ECM, VECM, ...).

The problem with early co-integration testing was the **weak power of the tests**, i.e. a small likelihood to reject the null hypothesis. This seems to be due to the (short) length of the series. Implementing panels rather than extending the length of the series has numerous advantages. (Pedroni 2004)

Co-Integration in Panels 2

$$y_{i,t} = \alpha_i + \delta_i t + \beta_i X_{i,t} + e_{i,t} \tag{4}$$

- $y_{i,t}$ and $X_{i,t}$ are $I(1)$ for each panel member i.
- Fixed effects α_i, deterministic trends δ_i and co-integration vectors β_i vary between members.
- H_0: None of the panel members are co-integrated.
- $H_{A,1}$ All members are co-integrated with the same co-integration vector. (Kao 1999)
- $H_{A,2}$: All members are cointegrated (the co-integration feature has to be the same for all β_i). (Pedroni 2004)
- $H_{A,3}$: A significant portion of the individuals are co-integrated. (Pedroni 2004)

Co-Integration in Panels: Special Cases

For homogenous slope parameters β and strictly exogenous regressors, **no co-integration** is asymptotically equivalent to **raw panel unit root tests**.

For homogenous slopes and partially endogenous regressors, critical values for the unit root tests have to be adjusted.

For heterogenous slope parameters, residual-based test for co-integration must not be based in **pooling** test statistics.

$$(y_{i,t}, X'_{i,t}) = z'_{i,t} = z'_{i,t-1} + \xi_{i,t} \tag{5}$$

$$\xi'_{i,t} \equiv (\xi^{y}_{i,t}, \xi^{x}_{i,t}) \tag{6}$$

Co-Integration: Assumptions

Assumptions: Let $B_i\Omega_i$ be a Brownian motion with asymptotic variance Ω_i.

A **Brownian motion** is a process with small, random movements in any dimension.

- $\frac{1}{\sqrt{T}} \sum_{t=1} [Tr] \to B_i\Omega_i$ **Central Limits Theorem** hold for each individual member.
- $E[\xi_{i,t}\xi'_{js}] = 0 \quad \forall i \neq j, s \neq t$ Central Limit Theorem holds in the **cross-sectional dimension**. \end{align*}

Co-Integration with homogenous slopes (Kao)

$$
\begin{aligned}
x_{it} &= x_{i,t-1} + e_{i,t} \\
y_{i,t} &= y_{i,t-1} + v_{i,t} \\
y_{i,t} &= \alpha + \beta x_{i,t} + u_{i,t} && (7) \\
\hat{u}_{i,t} &= \rho \hat{u}_{i,t-1} + \epsilon_{i,t} && (8)
\end{aligned}
$$

Dickey-Fuller Test:

$$
\begin{aligned}
\hat{\rho} &= \frac{\sum_{i=1}^{N} \sum_{t=2}^{T} \hat{u}_{i,t} \hat{u}'_{i,t}}{\sum_{i=1}^{N} \sum_{t=2}^{T} \hat{u}_{i,t}^2} \\
DF &= \sqrt{N} T(\hat{\rho} - 1)
\end{aligned}
$$

Augmented Dickey Fuller Test:

$$
\begin{aligned}
\hat{u}_{i,t} &= \rho \hat{u}_{i,t-1} + \sum_{j=1}^{p} \phi_j \Delta \hat{u}_{i,t-j} + e_{i,t,p} \\
ADF &= \frac{\hat{\rho} - 1}{s.e.(\hat{\rho})}
\end{aligned}
$$

where the number of lags p is chosen such that the errors $e_{i,t,p}$ are serially uncorrelated.

Co-Integration with heterogenous slopes (Pedroni) 1

$$
\begin{aligned}
x_{it} &= x_{i,t-1} + e_{i,t} \\
y_{i,t} &= y_{i,t-1} + v_{i,t} \\
y_{i,t} &= \alpha + \beta x_{i,t} + u_{i,t}
\end{aligned} \tag{9}
$$

Let Ω_i be the asymptotic covariance matrix for each member, Σ_i contemporaneous covariance among the components of $\xi_{i,t}$ for a given cross section i, and Γ_i the dynamic covariance between components of $\xi_{i,t}$. Ω **gives the long run co-integrating relationship (if there is one)**.

$$\Omega_i = \lim_{T\to\infty} E[\frac{1}{T}\sum_t^T \xi_{i,t} \sum_t^T \xi'_{i,t}] \tag{10}$$

$$\Omega_i = \Sigma_i + \Gamma_i + \Gamma'_i \tag{11}$$

$$\Sigma_i = \lim_{T\to\infty} \frac{1}{T}\sum_t^T E[\xi_{i,t}\xi'_{i,t}] \tag{12}$$

$$\Gamma_i = \lim_{t\to\infty} \frac{1}{T}\sum_{k=1}^{T-1}\sum_{t=k+1}^{T} E[\xi_{i,t}\xi'_{i,t}] \tag{13}$$

Co-Integration with heterogenous slopes (Pedroni) 2

Fortunately, $\hat{\Omega}_i$ can be efficiently estimated after de-composition.

Retrieve $\hat{xi}_{i,t} = (\hat{v}_{i,t}, \hat{\epsilon}_{i,t})$ from auto-regressions for each member i:

$$x_{i,t} = \rho_i x_{i,t-1} + \epsilon_{i,t}$$
$$y_{i,t} = \rho_i y_{i,t-1} + v_{i,t}$$

$$\hat{\Omega}_i = \hat{\Sigma}_i + \hat{\Gamma}_t + \hat{\Gamma}'_i$$
$$\hat{\Omega}_i = \frac{1}{T}[\sum_t^T \hat{xi}_{i,t}\hat{xi}_{i,t} + \sum_s^{k_i}(1 - \frac{s}{k_i+1})\sum_{t=s+1}^T(\hat{\xi}_{i,t-s}\hat{\xi}'_{i,t} + \hat{\xi}_{i,t}\hat{\xi}'_{i,t-s})] \tag{14}$$

Co-Integration with heterogenous slopes (Pedroni) 3

For the estimation, one runs the co-integration regression for each panel member, and calculates the lower triangular decomposition $\hat{L}_{11,i} = (\hat{\Omega}_{11,i} - \hat{\Omega}^2_{21,i}/\hat{\Omega}_{22,i})$.

Then one runs the residual regression for each panel member and calculates the group mean statistics (H_0: no cointegration).

$$
\begin{aligned}
y_{i,t} &= \alpha + \beta x_{i,t} + u_{i,t} \\
\hat{u}_{i,t} &= \rho \hat{u}_{i,t-1} + e_{i,t}
\end{aligned}
\tag{15}
$$

Furthermore, one calculates the auxiliary statistic $\lambda_i = (\hat{\sigma}^2_i - \hat{s}^2_i)$.

$$
\begin{aligned}
\hat{\sigma}^2_i &= \frac{1}{T}[\sum_t^T \hat{e}^2_{i,t} + 2\sum_s^{k_i}(1 - \frac{s}{k_i+1})\sum_{t=s+1}^T \hat{e}_{i,t}\hat{e}'_{i,t-s}] \\
\hat{s}^2_{i,t} &= \frac{1}{T}\sum_t^T \hat{e}^2_{i,t}
\end{aligned}
$$

Co-Integration with heterogenous slopes (Pedroni) 4

One can then calculate the group mean Z-statistics.

$$\tilde{Z}_{\hat{\rho},N,T} = \sum_{i}^{N} \frac{\sum_{t}^{T} (\hat{u}_{i,t-1} \Delta \hat{u}_{i,t} - \hat{\lambda}_i)}{\sum_{t}^{T} \hat{u}_{i,t-1}^2} \tag{16}$$

$$\tilde{Z}_{t,N,T} = \sum_{i}^{N} \frac{\sum_{t}^{T} (\hat{u}_{i,t-1} \Delta \hat{u}_{i,t} - \hat{\lambda}_i)}{\sum_{t}^{T} \frac{1}{\hat{L}_{11,i}^2} \hat{u}_{i,t-1}^2} \tag{17}$$