Wage Inequalities under Real Competition

A classical model for the equalization of ever-renewed inequalties and the shape of the wage curve.

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May 2023

Abstract

In classical political economics, the common wage level and wage inequality are results of the same economic and political processes. We trace Smith (1999)'s and Marx (1993a)'s treatment of wage inequalities and find that both describe processes of turbulent equalization, where labor's and capital's dynamics are intimately interwoven. The literature on real competition and wage inequality (Botwinick 1993; Shaikh 2020; Mokre and Rehm 2020) poses (1) that competition links profit rates and wage growth, (2) both behave as turbulent processes, and (3) the dynamics contain both determinate and stochastic components. We model the "*constant equalization of ever-renewed inequalities*" (Marx 1993a, 197–298) where "*the competition among workers is only another form of the competition among capitals*" (Marx 1993b, 651) and find an empirical expression in drift-diffusion models.

We present a multi-sector model of turbulent wage growth and persistent wage inequality. In real competition, when a sector realizes above-average profit rates on new capital and this induces accelerating investment streams. Increased labor demand as well as higher profit rates shift the limits to wage growth. The subsequent fall below the average in later periods, which is characteristic for turbulent equalization, also translates into wage decreases. Finally, we formalize a wage growth model in stochastic differential equation form of a Cox, Ingersoll, and Ross (1985) - style drift - diffusion process, and derive analytically the corresponding cross - sectional distribution parameters (Fischer 2018). When we apply the model to US wage growth data (1998 – 2018), the estimation explains about 93 % of the wage distribution below the top percentile, and 86 % of total sample inequality.

1 Introduction

Inequality is the defining feature of capitalist economies and capitalist economic dynamics. The ubiquitous hunt for higher profit rates and wages follows unequal expected rates and creates new disparaties. Thus, inequality is a core interest of classical political economics, even more so as the issue increased in prominence among neo-classical and heterodox economists in recent years.

Income inequalities exist along many dimensions, for example the distribution of national income between profits and wages, the gender and racial wage gaps, education premiums or regional disparities. In this paper, we emphasize the dimension of deviations from the prediction that "*equally qualified workers who labour under similar working conditions should tend to receive roughly equal compensation*" (Botwinick 1993, 1), which we generalize to the wage curve. We discuss the classical political economics treatment of wage inequality to derive a corresponding model of turbulent wage growth dynamics and resulting wage level distributions. We develop a stylized empirical model and present a Bayesian estimation strategy for illustration.

The equalization of profits and wages is a fundamental aspect addressed in the works of Adam Smith and Karl Marx concerning competition. Both discuss tendencies of equalization as well as persistent inequalities. Smith (1999, 163) emphasizes the establishment of new firms and their demand for already-employed workers as a source of wage differentials. Similarly, Marx highlights the significance of capital's laws of motion in generating inequalities among workers, which reflect their relationships with one another. Like Smith, Marx views the general wage level as an outcome of conflicting bargaining (Marx 1990, 3:758–60), he argues that wages increase when capital accumulates faster than the labor force grows (Marx 1990, 3:771). In between-industry competition, this predicts unequal accumulation and asynchronously rising wages where profit rates on new capital are above the general level in "*ever-renewed inequalities*". (Marx 1993a, 297–98) Both Smith and Marx describe wage growth as turbulent processes of simultaneous equalization and dis-equalization, set in motion by investment dynamics, which create inequality between otherwise equal workers expressed as wage disparities from the general level.

Neo-classical models of wage inequality stand in sharp contrast to this approach. The debate on the issue arose again in the 1980s, when empirical investigations showed abrupt changes in the wage distribution, an increasing share of low-wage jobs and growing disparity between low and high wages. (Lemieux 2007, 22) Within the neo-classical framework, corresponding extensions of competitive wage setting explain deviations from equal wages for equal quality labor by non-perfect competition settings of micro-economic optimization. These include efficiency wages in shirking, labor turnover or adverse selection models (Yellen 1984, 203; Bowles 2006, 267–98) [1] , or frictions from economic transformation pushing the labor market off equilibrium in the skill-biased technological change (SBTC) model. (Krueger 1993; Acemoglu 2002) Another literature focus on changes in wage-setting institutions. (Freeman and Katz 1995) In this category of models, wage inequalities stem from non-competitive institutions like dual labor markets (Doeringer and Piore 1971), monopoly (Galbraith 1998) and monopsony (Ashenfelter et al. 2021); or institutional changes such as de-unionization (Card 1992; Ahlquist 2017) and minimum wage laws (DiNardo, Fortin, and Lemieux 1996; Lee 1999).

Both classes of models identify deviations from perfect competition as the reason for wage inequality, and take perfect competition as their point of departure. Botwinick (1993) calls this the "*historical impassé*", as researchers would have to either model labor markets fundamentally at odds with empirical (unequal) reality, or to give up on competition to explain wage dynamics. To break the impassé, he traces the interaction of competitive dynamics and wage inequality back to Marx' Grundrisse: "*The competition among workers is only another form of the competition between capitals.*" (Marx 1993b, 651)

In the spirit of Smith's and Marx' treatment of wage differentials, the real economic analysis literature seeks to understand wage inequality as the product of interactions between workers and firms within institutional wage-setting regimes. It builds on Shaikh (1980)'s model of real firm competition, where turbulent equalization of profit rates on new capital give rise to persistent inequalities. (Weeks 2001; Shaikh 2008) Real competition models of wage inequality combine institutions of wage bargaining with the competitive dynamics behind firms' ability and willingness to pay. (Botwinick 1993; Watson 2002; Mokre and Rehm 2020; Shaikh 2020)

The concept of turbulent equalization of profit rates (Shaikh 2008) and wages (Mokre and Rehm 2020) finds a modeling equivalent of combined turbulent and persistent processes in the econophysics of inequality literature (Gibrat 1931; Adrian Drăgulescu and Yakovenko 2001; Fischer 2018; Shaikh 2020). Ragab (2014) provides an agent-based real competition model, while Shaikh and Jacobo (2020, 10) use turbulent drift-diffusion equation modeling to explain the striking statistical regularity of wage income converging to an exponential or a Gamma distribution.

Our empirical investigation stands in the same real competition - econophysics tradition, we derive the distribution of wage levels from the turbulent dynamics of wage increases. The model captures the interlinked turbulent tendencies of real firm competition, which translates into the labor market via limits to wage increases and labor demand, and the idiosyncratic dynamics of labor supply. At this level of abstraction, the interaction of between-industry and within-industry inequalities is expressed in simultaneous equalization and diffusion of wages, which give rise to a stable distribution. We bring together the real competition literature on firms' limits to wage increases with the literature on turbulent wage dynamics, and present a unified model of wage inequality under real firm competition. We furthermore present a novel Bayesian estimation of drift-diffusion equations and fit a simple empirical model to US wage and wage growth data 1987-2018.

[1] "*The theories reviewed above are neoclassical in their assumption of individualistic maximization by all agents*" (Yellen 1984, 203)

As in Marx (1993b), we take the interaction of firm and labor market competition as a point of departure and note that wages participate in turbulent equalization as shown in Mokre and Rehm (2020). After reviewing the classical, heterodox and econophysics literature in section 2, we derive our theoretical model of turbulent competition and wage inequality in section 3. In section 4 we use a novel Bayesian estimation technique of drift-diffusion models and derive the corresponding cross-sectional distribution of wages. Section 6 presents the results and the fit of the model, which explains 86 % of overall wage inequality, and 92 % of the inequality in the bottom 99 percentiles. In section 7 we summarize the findings, align them with the literature, and propose extensions of the model .

2 Literature

In classical political economics, the debate on wage inequality is a small but integral part of wage theory, dis-equalization reveals how wages are formed and set in motion equalizing tendencies. This is a key difference to neo-classical wage theory where wage levels are the result of competition while inequalities are the consequence of obstacles to competition, ie. wages and wage disparities have different and antithetic explanations.

Smith was credited with constructing the first comprehensive theory of wages (Lapides 1998, 31; Schumpeter, Schumpeter, and Perlman 1997, 256) In Chapter 8 and 10 of "The Wealth of Nations", Smith describes the wage level as a result of economic as well as political laws. Where the wage level rises above the absolute physical minimum for reproduction ("*evidently the lowest which is consistent with common humanity*" (Smith 1999, 101)) it follows the increase in output and resulting increases labor demand.

"*The demand for those who live by wages, therefore, necessarily increases with the increase of the revenue and stock of every country, and cannot possibly increase without it. [...] The demand for those who live by wages, therefore, naturally increases with the increase of national wealth, and cannot possibly increase without it. It is not the actual greatness of national wealth, but its continual increase, which occasions a rise in the wages of labour.*" (Smith 1999, 102–3)

At the same time, Smith describes wage-setting as a political, conflictive and bargaining process where capital is at an advantage and supported by political powers in parliament. [2]

"*The workman desires to get as much, the masters to give as little as possible. The former are disposed to combine in order to raise, the latter in order to lower the wages of labour. It is not, however, difficult to foresee which of the two parties must, upon all ordinary occasions, have the advantage in the dispute, and force the other into a compliance with their terms. The masters, being fewer in numbers, can combine much more easily; and the law, besides, authorizes, ar at least does not prohibit their combinations, while it prohibits those of workmen. We have no acts of parliament to lower the price of work; but many against combining to raise it.* " (Smith 1999, 98–99)

In summary, the wage level is driven by demand for labor, which is in turn determined by capital accumulation. Mobility of labor between employments and the competition of employers for labor set and equalize wages. This concrete process however takes place a bargaining over contracts, with collective organization and the legal-political framework as key factors.

This is the point of departure for Smith's treatment of wage disparities. In Chapter 10, he lists five sources of unequal wages.

"*The five following are the principal circumstances which, so far as I have been able to observe, make up for a small pecuniary gain in some employments, and counterbalance a great one in others: first, the agreeableness or disagreeableness of the employments themselves; secondly, the easiness and cheapness, or the difficulty and expense of learning them; thirdly, the constancy or inconstancy of employment in them; fourthly, the small or great trust which must be reposed in those who exercise them; and, fifthly, the probability or improbability of success in them.*" (Smith 1999, 143)

[2] smith finds examples pro-capitalist legislation throughout Chapters 8 and 10, including an accusation of King George III: "*This law is in favour of the workmen: but the 8th of George III is in favour of the masters.*"

A key moment in persistent inequalities is differential costs of reproducing skill for a certain kind of labor.

"*A man educated at the expense of much labour and time to any of those employments which require extraordinary dexterity and skill, may be compared to one of those expensive machines. [. . .] The difference between the wages of skilled labour and those of common labor is founded upon this principle.*" (Smith 1999, 145)

By describing unequal wages between types of labour, Smith reinforces the idea of equal pay for equal labor. The processes responsible for inequality are the same ones driving equalization within the same occupation; the mix of skills and disparities of cost to reproduce those skills set up the basic structure of the wage curve. From this point of departure, Smith finally describes dynamic processes that create inequalities within occupations and at the same time set in motion processes of equalizations. They are driven by capitalist competition and technological innovation expressed in the establishment of new firms as competitors for labor, a ubiquitous and recurring event in all capitalist economies. This describes a simple turbulent model, which in turn gives rise to a persistently unequal and potentially stable distribution of wages.

"*Where all other circumstances are equal, wages are generally higher in new than in old trades. When a projector attempts to establish a new manufacture, he must at first entice his workmen from other employments by higher wages than they can either earn in their own trades, or than the nature of his work would otherwise require, and a considerable time must pass away before he can venture to reduce them to the common level.*" (Smith 1999, 163)

It is also noteworthy that Smith has these wage rates return to some common level once a trade is established. Even in this simplest model, the succession of dis-equalization and equalization, expansion and compression of the wage distribution provides the basis for a turbulent process of wage-setting. Considering that for Smith, new firms are founded in search of above-average profits, he model also already incorporates the intricate link between profit rates and wages.

Marx's theory of wages is developed in sharp distinction from post-Ricardo interpretations, especially as a criticism of the post-Ricardo wage fund/labor fund dogma. While Marx criticizes Smith's confusion of labor and labor power, the model of wage setting and wage inequalities in Capital takes his argument as a point of departure. Consistent with the labor theory of value, he normal wage level fluctuates around the value of the commodity labor power, which is given by the reproduction cost for the worker, their skill, the next generation of workers, and their participation in normal cultural life (the last aspect is determined socially). The normal wage levels for different qualities of labor fluctuate with accumulation and the reserve army of labor.

"*Taking them as a whole, the general movements of wages are exclusively regulated by the expansion and contraction of the industrial reserve army, and this in turn corresponds to the periodic alternations of the industrial cycle.*"(Marx 1990, 3:790)

As in Smith, Marx has the general wage level determined by political struggle as well as economic laws of capital accumulation and competition. Changes in workers' wages, ie. wage growth, come from capitalist accumulation: "*To put it mathematically: the rate of accumulation is the independent, not the dependent variable; the rate of wages is the dependent, not the independent variable.*" (Marx 1990, 3:770) After capital has been accumulated through unpaid labor, its valorization becomes the new ecnomic imperative. When he speed of accumulation outpaces labor supply (the industrial army and the reserve army in sum), wages follow part.

"*If the quantity of unpaid labour supplied by the working class and accumulated by the capitalist class increases so rapidly that its transformation into capital requires an extraordinary addition of paid labour, then wages rise and, all other circumstances remaining equal, the unpaid labour diminishes in proportion.*" (Marx 1990, 3:771)

The link between Marx' wage theory and wage differentials lies in inter-industrial differences in accumulation. Capital moves towards industries with above-average profit rates on new investment, where the increased supply and price competition subsequently drives the profit rate down below the general rate in a turbulent fashion. (Shaikh 1980) Consequently, workers encounter differential wage growth between industries.

"*The economic fiction we have been dealing with confuses the laws that regulate the general movement of wages, or the ratio between the working class - i.e. the total sum of labour-power - and the total social capital, with the laws that distribute the working population over the different spheres of production. If, for example owing to a favourable conjuncture, accumulation in a particular sphere of production becomes especially active, and profits in it being greater than the average profits, attract additional capital, then of course the demand for labour rises, and wages rise as well.*" (Marx 1990, 3:791–92)

As wage growth differs between industries and follows the turbulent pattern of profit rates in competition, the movement of workers resembles the mobility of capital. In the Grundrisse, Marx discusses capitalist competition as the expression of free capital accumulation is imposed on all economic processes in capiatalism: "*The reciprocal compulsion which the capitals within it practise upon one another, on labour etc. (the competition among workers is only another form of the competition among capitals)*" (Marx 1993b, 651) In Volume 3 of Capital, Marx specifies how capitalist competition instigates workers' competition.

"*If capitals that set in motion unequal quantities of living labour produce unequal amounts of surplus-value, this assumes that the level of exploitation of labour, or the rate of surplus value, is the same, at least to a certain extent, or that the distinctions that exist here are balanced out by real or imaginary (conventional) grounds of compensation. This assumes competition among the workers, and an equalization that takes place by their constant migration between one sphere of production and another.*" (Marx 1993a, 275)

The competition of workers does not only produce wage inequalities for the same quality of work. While increased capital acceleration in above-average profit rate industries put downward pressure on prices through increased competition and supply (with the corresponding effect on wages), the speed at which workers can follow the movements of capital and valorize it amplify equalization.

"*This constant equalization of ever-renewed inequalities is accomplished more quickly, (1) the more mobile capital is, i.e. the more easily it can be transferred from one sphere and one place to others; (2) the more rapidly labour-power can be moved from one sphere to another and from one local point of production another.*" (Marx 1993a, 298)

Finally, it is crucial to note Marx's argument about obstacles of equalization. In the introduction to Volume 3 of Capital's Chapter 8, Marx argues that inter-industrial wage differentials are mainly carried by different mixes of labor qualities: One industry will employ more complex labor, and accelerated accumulation in that industry will push wages more than in one with a higher share of simple labor employed. In the same segment, Marx notes that a general tendency of equalization and obstacles to that process exist at the same time.

"*And even though the equalization of wages and working hours between one sphere of production and another, or between different capitals invested in the same sphere of production, comes up against all kinds of local obstacles, the advance of capitalist production and the progressive subordination of all economic relations to this mode of production tends nevertheless to bring this process to fruition. Important as the study of frictions of this kind is for any specialist work on wages, they are still accidental and inessential as far as the general investigation of capitalist production is concerned and can therefore be ignored.*" (Marx 1993a, 241–42)

Taken together, Marx' argument on wage setting in Capital and the Grundrisse provide a system of turbulent equalization. Smith too presents a general economic intuition (perpetual innovation) for ever-renewed inequalities in wages and simultaneous economic drivers of equalization. Marx embeds a similar intuition into a general analysis of accumulation. While he notes that the frictions to equalization do not concern the theory of capital accumulation, they are crucial to explain the wage curve - what he calls a *specialist work on wages*. Simultaneous diffusion and equalization, where equalization is not infinitely fast (ie. with obstacles to equalization), can be formulated in stochastic differential equations (SDEs). In certain simple forms, a stable distribution of wage levels can be derived; the economic models of Smith and Marx can be expressed mathematically to derive the form of the wage curve.

The literature on real economic analysis, and more specifically on real competition (Shaikh 1980) theorize and demonstrate the turbulent equalization of profits and investment. Following Marx's assertion, that capital through political struggle forces its competitive dynamics on all economic categories including labor and wages, the same literature corroborated the turbulent equalization of wages. (Ragab 2014; Mokre and Rehm 2020; Shaikh 2020) In real competition, firms make the decision to reinvest in their facilities, or cross-invest into different industries, by searching the highest expected profit rate on new capital ("regulating profit rate"). Industries with a reproducible production technology which offers an above-average profit rate on new capital will experience accelerated investment, those industries with below-average rates will experience deceleration.

Profit rates on new capital descent directly from the profit margin, the difference between production cost and the industrial price for a good. In order to push out competition, the lowest-cost capital in an industry can decrease prices such that they still make a positive profit, while the closest competitors do not. The consequence of more intense competition is falling prices and subsequently, a lower regulating profit rate. Thus, as investment accelerates and competition increases, the before-above-average regulating profit rate will fall below the mean, while the earlier abandoned below-average industry rises again, and becomes an attraction point for investment again. This process never converges to equal equilibrium, as technological progress and competitive attacks are ubiquitous. Rather, they induce cycles after cycles of turbulent equalization.

Turbulent equalization rules the most important aspects of economic life, as the logic of capital accumulation is reproduced in all subsections of the production process. The normal (in Smith: common) relationship then provide the gravitational center around which profits, prices or wages turbulently equalize.

"*Classical theory postulates that competition turbulently equalizes prices for equivalent types of products, wage rates for equivalent types of labor, and profit rates for equivalent risks. [. . .] If real wages are higher in some firms and lower in others, the supply of labor seeking jobs increases in the first set and decreases in the second. At this level of abstraction, wage rates will be turbulently equalized across firms and hence across industries.*" (Shaikh 2016, 749–50)

Shaikh's turbulent equalization of wages and profits are modeled at a lower level of abstraction than Smith's wage inequalities due to new investment or even Marx's equalization and dis-equalization. His model grounds equalizing tendencies in inequalities, which are in turn brought upon by firms' equalizing behavior.

The real economic analysis approach to profit rate and wage inequalities posits that some extent of inequality is brought not by an absence of competitive equalization, but by its concrete dynamics. In "Persistent Inequalities", Howard Botwinick models mechanical links between profit rate and wage rate equalization in competition. He furthermore shows how a process of turbulent profit rate and wage equalization brings upon persistent between-industry inequalities.

"*Based on over one hundred years of empirical evidence, a viable theory must be able to explain how substantial wage differentials among comparable workers can quite obviously persist under highly competitive conditions.*" (Botwinick 1993, 7–8)

Botwinick explains the competitive mechanisms that produce persistent wage inequalities. The first is the systematic de-skilling of workers through the unemployed reserve army. (Botwinick 1993, 100; Braverman 1974, 179) The expulsion of workers from production is a permanent and repeating feature of capitalist competition: Defeated firms disappear with corresponding job losses, technological progress within the firm replaces workers on old contracts and with old tasks. Job loss is here the consequence of a turbulent process. Unemployed workers would mostly be re-employed at a lower wage and not using all of the productive skill they attained over time. This increases inequality between workers of the same education and experience, as some re-enter employment at the bottom of the wage distribution.

Botwinick's second transmission mechanism are the limits to wage bargaining. Wages are the subject of active negotiation, with wage increases rather than limits the subject of bargaining. All other things equal, workers can win higher wage gains when capitalists are able to pay more or willing to pay more. This can be formalized as (1) positive profits per worker and (2) a positive cost differential of the firm to the closest competitor per worker.(Botwinick 1993, 184–88) Both limits have one turbulent component as well as one that is persistently different between industries. Like Smith's and Marx' models of wages in capitalist competition, the combination of turbulently equalizing and persistently different factors in wage-setting provide economic intuition for a model of simultaneous diffusion, equalization and obstacles to equalization.

The econophysics literature has discussed stochastic models of income growth going back as early as the 19th century. Pareto (1897) might have been first to observe that (top) income distributions follow a power law, with Gibrat (1931) providing multiplicative stochastic processes as an explanation. (Adrian Drăgulescu and Yakovenko 2001, 213) Later discussions (Adrian Drăgulescu and Yakovenko 2001; Banerjee, Yakovenko, and Di Matteo 2006; Shaikh, Papanikolaou, and Wiener 2014) find that the exponential distribution provides a reasonable fit to the lower 95 % - 98 % of the income distribution, which they identify with wage income. Adrian Drăgulescu and Yakovenko (2001) and Banerjee, Yakovenko, and Di Matteo (2006) report that a Gamma distribution better approximate wage income data, however in a near-exponential parametrization. [3] Gabaix (2009) and Fischer (2018) demonstrate how to use drift-diffusion models to derive persistent wage inequality, where they combine an equalizing (mean-reverting) drift term with a stochastic diffusion process.

In this paper, we combine the CPE model of firm competition-derived limits to wage increases (Botwinick 1993) and fit an econophysics drift-diffusion models (Shaikh 2020). We do so by combining turbulent flows of labor with turbulent processes in wage bargaining, combining Botwinick (1993)'s and Shaikh (2020)'s approaches. The simultaneous and connected turbulent equalization processes of profit rates and wage rates provide the link in our theoretical model and empirical strategy.

3 Model

Wages are set in bargaining between workers and capitalists. The subject of bargaining is wage increases more often than wage levels: In ongoing employment, there is a strong and causal relationship between one period's pay and the next one. Also, workers who switch workplaces (both in direct poaching or worker-induced career changes) consider the remuneration differentials, as do employers in their wage offers. When unemployed workers act as "wage takers", the literature finds their wage offer decline with length of unemployment (and, supposedly, their bargaining power). However, their offered wage is an institutional result of bargaining over wage increases, but also corresponds to pre-unemployment wages. (Christensen 2002)

In our model, we abstract from individual characteristics and even demographic groups, to describe the fundamental process of *shaping the wage curve*. This does not not assume away pay differential between social groups, eg. men and women, or white and racially marginalized workers. Botwinick (1993) proposes to study gender and racial pay gaps as being positioned along a common wage curve, which is produced by the dynamics of capital and labor competition. Indeed, Shaikh, Papanikolaou, and Wiener (2014) found that the shape of the wage distribution is the same within these groups (however with different means).

Thus, we study how the turbulent dynamics of profits, investment and employment shape the wage curve on a level of abstraction where equalization and individual-level diffusion are explicitly modeled. While we argue that persistent between-industry and between-occupation pay differentials are crucial in that process, we do not explicitly model the underlying processes. That dynamic does not depend on the characteristic of one industry, occupation, the extent of gender discrimination or racial segregation, but on the interlinked processes of firm and worker competition.

[3] Banerjee, Yakovenko, and Di Matteo (2006) prefer the exponential form, as it does not have zero density at zero wages. Adrian Drăgulescu and Yakovenko (2001) prefer the exponential over the Gamma distribution for simplicity. In our application, ie. the wage income of employed workers, zero mass at zero wages is not inappropriate. The restriction to employed workers and wage income corresponds to our modeling of employer-employee relations as an underlying cause for inequality. While A. Drăgulescu and Yakovenko (2001) model income with an exponential function as the result of a Boltzmann-Gibbs function, they do not distinguish between wage and non-wage income and consequently explicitly include zero income-observations. Use of the Gamma distribution to estimate wage income distribution was pioneered as early as in the 19th century by March (1898), and to US pre-tax income by Salem and Mount (1974). As Kleiber and Kotz (2003) cite, the theoretical Gini coefficients from a Gamma distribution fall into more realistic boundaries than those of competing functions. While the Gamma distribution is consistently out-performed by distributions with more parameters, McDonald (1984) find that it provides a much better fit to income data than other two-parameter distributions.

3.1 Wage bargaining under real capitalist competition

Bargaining results are restricted by behavioral limits, namely workers' credible threats, capitalists' willingness to pay, and finally capitalists' ability to pay. The last is subject to law-like economic processes which firm competition between and within industries induce. The maximum wage increase is such that the the regulating capital's profit margin per worker remains strictly positive; it can be written as the product of regulating profit rates r^* and the capital-labor ratio K/L: $\frac{m}{L/Q} = r^* \frac{K}{L}$.(Botwinick 2018, 213) Regulating profit rates differ between industries at any point in time, but participate in turbulent equalization (ie. an industrial regulating profit rate will be above and below the cross-sectional average in alternating "*fat and lean years*, Shaikh (2008)). Capital-labor ratios on the other hand are persistently different depending on the technologies used in the production process.

Competition within an industry dictates a second limit for regulating capitals (ie., the firms with the highest reproducible profit rate on new capital) such that total unit cost do not rise above the one of the subdominant capital (the closest competitor). (Botwinick 2018, 179–95) If we compare two industries with the same cost differentials per unit labor requirement $\frac{(k^*-k^s)/l^*}{k^*/l^*}$, ie. the same intensity of competition, the difference in limits to wage growth is equivalent to the ratio of total cost-labor cost ratios $\frac{(k/l)_A}{(k/l)_B}$. (Botwinick 2018, 222) The limits for regulating capitals deserve special attention, as "*these capitals represent the most competitive conditions that can be reproduced, they essentially act as the practical standard for the industry as a whole*" (Botwinick 2018, 200)

Non-regulating capitals have to follow regulating capitals in goods prices to retain market shares, but also in wages to prevent labor poaching. They have tighter limits to wage increases, primarily to keep a positive profit margin per worker, while keeping prices fixed. (Botwinick 2018, 257) While rising wages in regulating capitals mean that non-regulating firms follow in the same direction, increases in these firms fall short only to widen *inter-industry wage differentials.* (Botwinick 2018, 268)

Thereby competition between and within industries sets the limits to wage increases within firms and sectors. These limits change between years, and each has one turbulent and one persistently differential component. The competitive limits to wage growth induce a turbulent dynamic in wages and in wage-seeking labor supply.

3.2 Profit rates and wages participate in turbulent equalization

Both regulating profit rates and cost differentials participate in turbulent equalization (Mokre 2021). This produces "crossing over" patterns, where industrial incremental rates of return intersect frequently and gravitate around some common value (Shaikh 2008). Tescari and Vaona (2014) provide an econometric interpretation in which each industry's deviations from the cross-sectional mean (eg. $\dot{r} = r - \bar{r}$) are zero on average over time, and have no discernible time trend.

In real competition, higher regulating profit rates attract investment and increase competition. Increased competition translates into smaller within-industry cost differentials as well as falling regulating profit rates, ie. lower limits to wage increases. When such an industry consequently realizes below-average profit rates on new capital, the investment dynamic is reversed, without ever reaching steady state equilibrium. [4] The general profit rate on new capital constitutes the center of gravitation for the turbulent process.

Wage increases show a similar turbulent pattern above and below the cross-sectional mean (eg. $\dot{w} = w - \bar{w}$). Furthermore, there is a significant and substantial impact of regulating profit rates on wage increases. (Mokre and Rehm 2020) Regulating profit rates are attractors to wage increases, profit-rate induced accelerations of investment are attractors to employment. Labor is mobile in a similar fashion like capital, with increasing wage increases, accelerated inflow of workers, subsequently falling wage increases and deceleration of employment growth. (Shaikh and Jacobo 2020)

[4]"*So great an accession of new business to be carried on by the old stock must necessarily have diminished the quantity employed in a great number of particular branches, in which the competition being less, the profits must have been greater*" (Smith 1999, 134–35)

3.3 Hiring and Firing give rise to wage diffusion

Labor market dynamics are expressed on the individual level by firms hiring and firing workers, or changing employment conditions. These dynamics are defining features of capitalist production, as Smith (1999, 163–64) notes, the establishment of new and more profitable production conditions fuel both technological change and product innovation.

Hiring almost always means a wage increase for the affected individual, whether compared to previous unemployment or a previous job. Firing on the the other hand does not only immediately reduce an individual worker's income, more often than not it also means that when re-hired, they earn less than before because they are not employed (and not paid) at the same skill level of their old job. (Braverman 1974, 179) The systematic expulsion from employment due to technological change, be unemployment temporary or not, de-skills the affected workers and reduces their wages. Hirings (due to capital accumulation) and firings (due to technological change or elimination of firms from competition) occur all the time, and at the same time. The simultaneous processes give rise to the ubiquitous tendency towards wage diffusion. Bargaining, ie. changing the conditions of employment is a process of constant diffusion as noted in subsection 3.1.

3.4 Skilled labor forms differential centers of gravity in equalization

Wages participate in turbulent equalization because labor is mobile and workers behave in a wage-seeking fashion. When higher wage increases are achieved in an industry, more workers will fill offered jobs. Furthermore, if one firm in the industry is paying better than the competition, they will be more successful in poaching workers from other firms. However, wages equalize for equivalent jobs and "skilled labor" usually earns higher wages corresponding to the cost of producing and reproducing that skill. Different "costs of skilling" act as different centers of gravitation in turbulent equalization. (Botwinick 1993, 11; Shaikh and Glenn 2018, 17) [5]

In classical political economics, the between-occupation dimension of wage inequality is modeled as a result of differential costs to re-skill. (Shaikh and Glenn 2018) Labor is bought by firms in different qualities, some skill applicable in production constitutes the use value of labor as a commodity, ie. the reason why firms buy it, the reason why labor as a commodity enters the market. Of course, labor power cannot be treated like any other commodity in the labor theory of value, as it can emit more value than is needed for its reproduction. Nevertheless, beginning with Marx, classical political economists pose that the cost of attaining productive skills enter into its price. [6]

When an industry experiences above-average regulating profit rates, or below-average competitive pressure on the regulating capital, the ability and willingness of firms to agree to higher wage gains increases. This accelerates labor supply, from other industries, from less profitable firms in the same industry, as well as from unemployment. Both regulating profit rates and "competitive space" are turbulent processes, industries experience alternating "*cycles of fat and lean years*". While the increase of labor supply in "fat years" is induced by the mobility of labor, higher wage limits go back to the dynamics of capital competition and profit rate equalization. While each industry and firm employs a different mix of occupations (Shaikh 2016, 750), these are not responsible for the turbulent process, which in turn induces the turbulent equalization of wage rates.

Thus, while different skill levels constitute different centers of gravitation (and the first cornerstone of wage inequality (Botwinick 1993, 12)), all qualities of labor participate in the same process of equalization. Mokre (2021) finds that all parts of the wage curve participate in turbulent equalization of wages, and the link between regulating profit rates and wage increases is present over the whole distribution as well.

[5] "*Classical theory postulates that competition turbulently equalizes prices for equivalent types of products, wage rates for equivalent types of labor, and profit rates for equivalent risks.*" (Shaikh 2016, 749)

[6] "*In order to modify the human organism, so that it may acquire skill and handiness in a given branch of industry, and become labour-power of a special kind, a special education or training is requisite, and this, on its part, costs an equivalent in commodities of a greater or less amount. This amount varies according to the more or less complicated character of the labour-power. The expenses of this education (excessively small in the case of ordinary labour-power), enter pro tanto into the total value spent in its production.*" (Marx 1990, 3:275–76)

Furthermore, the impact of within-industry competition on wage levels increases steeply for higher wages. In between-industry competition, classical political economics models often take differentials in the average wage levels to control for differential mixes of qualities of labor. (Shaikh 2016, 868) Not only do different levels of skill form different centers of gravitation, but the inequality-increasing diffusion effect is also higher for higher-paying occupations. The empirical model we present in subsection 4.1 explicitly accounts for the bigger volatility in poaching of higher-paid workers (by scaling the diffusion term with the square root of the wage level).

3.5 An example of two industries

As an example, think of two industries for basic and luxury goods. Let the luxury goods industry operate at persistently higher levels of capital intensity and a lower share of labor cost in total cost. Within both industries, firms realize different profit rates, corresponding to efficiency of their production process (the output-cost ratio) and prices. The lowest-cost capitalist can set prices just so low that they still realize a normal profit, while their less efficient competitors are gradually pushed out of the market. However, the lower prices also cut into the regulating capital's own profits. Price-setting depends on cost differentials between regulating and subdominant capital ("competitive space"), thus profit rates on new capital fall when intensity of competition rises.

After each period, capitalists earn profits and decide to reinvest them in either industry. If profit rates, using the newest available modes of production, are higher in the luxury industry, more capitalists will invest there.

We will assume three things about the labor market: First, accelerating investment does not go fully into replacing existing capitals, but also increasing facilities, and thus induce some employment surge. Second, there exist industry-specific labor markets which capitalists can access via coaxing from competitors. In coaxing, attacking firms offer wage gains, and "defenders" try and keep up. Third, there also exists one common labor market of workers who are not bound to one single industry and can be readily recruited from unemployment.

Workers in the luxury goods industry now face higher labor demand, at least some of it within the industry, which creates upward pressure on capitalists' willingness to increase wages. At the same time, higher regulating profit rates also increase capitalists' ability to pay. For the same organizational strength, bargaining will yield higher wage increases. Increased coaxing puts further upward pressure on wage gains. The opposite is true in the basic goods industry. This is consistent with the theoretical results in Botwinick (2018) as well as the findings of Mokre (2021).

As the industries' profit rates turbulently equalize, periods of above- and below average wage growth will alternate. However, capital intensity is higher, and the share of labor cost is lower in the luxury goods industry. For the same regulating profit rates, higher wage gains can be won (as we treat organizational strength uniform for now). Over time, this manifests in persistent and increasing industrial wage differentials. Mokre and Rehm (2020) provide empirical support for this hypothesis.

3.6 Turbulence links profits and wages

The gravitational center of profit rate turbulence acts as gravitating force for wage levels (not wage increases) to some cross-sectional mean. This is a determinate, mean-reverting drift process: Above-average wage levels go with smaller increases and vice versa. At the same time, turbulent profit rates on new capital pull wage increases towards persistently different capital-labor and labor cost-total cost ratios as attraction points in a stochastic diffusion process. It is noteworthy that in Mokre (2021), the impact of regulating profit rates on wage growth is strongest in extreme positive and negative growth (the lowest and highest deciles in a conditional quantile regression).

Common labor markets induce their own turbulent processes. Coaxing of skilled basic goods workers into the luxury industry leaves remaining workers in a better bargaining position with regards to inter-industry coaxing once regulating profit rates in the industry recover. At the same time, a larger labor force in the luxury goods industry can be pit against each other in face of downsizing, when regulating profit rates fall below the average and investment decelerates. This applies more to workers switching between existing employments, and less to workers who move through unemployment (and the implied loss of skill and wage gains), which is again consistent with Mokre (2021)'s finding that the impact of regulating profit rates is stronger in extreme deciles, ie. high gains and losses.

Workers participate in a turbulent process of employment and wage-setting. Limits to wage increases as well as labor demand are set on the firm- and industry-level by the competition of capital. The gravitational centers for the turbulent equalization differ between occupations. Nevertheless, the full extent of the process is captured by individual wage changes, and best measured on the worker level. We will use an appropriate drift-diffusion model (see Section 4), and individual wage date from the IPUMS-CPS (Section 5) to estimate the ensuing dynamics, and derive a corresponding stable distribution of wage levels from it. To capture the simultaneous dynamics of equalization and diffusion over the years, we normalize wages by their cross-sectional yearly mean (such that a wage of 1 gives the weighted average wage).

4 Method

The decomposition of the process in one determinate and one stochastic part can be analyzed using stochastic differential equations. In financial economics this is called drift-diffusion modeling (Fischer 2018), while physics knows the terms Fokker-Planck and Kolmogorov forward equations. Kolmogoroff (1931) We fit a Cox-Ingersoll-Ross (Cox, Ingersoll, and Ross 1985) drift-diffusion equation to US wage growth data to demonstrate the explanatory power of turbulent equalization for wage inequality. The model is sufficiently simple to provide economic interpretation of the growth process and wage distribution parameters. At the same time, it has the volatility of diffusion grow with the wage level, which corresponds to the classical political economics treatment of qualities of labor.

At the same time, there exist many drift-diffusion models with known analytical parameter transformations in the cross-section distribution, the Vasicek (1977) for example process gives a log-normal, and the Courtadon (1982) process an inverse Gamma distribution. A geometrical Brownian motion with jumps finally converges to a double-Pareto distribution of incomes. (Fischer 2018) While a discussion of ther advantages and shortcomings is fascinating (and has a rich history), in this paper we focus on one demonstration of feasibility.

4.1 Cox-Ingersoll-Ross model

Cox, Ingersoll, and Ross (1985) use drift-diffusion modeling for interest rates, as in Equation 1. In their model, a variable X grows such that it reverts to its mean μ with some speed θ, a tendency of equalization over time. At the same time, a Wiener process W moves every period following a Gaussian standard normal distribution, its impact on dX consists of propensity parameter σ and increases in $\sqrt{X}$.

The Cox-Ingersoll-Ross process has an asymptotically stationary cross-sectional Gamma distribution with parameters $\alpha = \mu\frac{2\theta}{\sigma^2}$ and $\beta = \frac{2\theta}{\sigma^2}$. The Gamma distribution is in use to fit income data since the 1920s (Amoroso 1925), among other reasons because the two parameters α and β can be interpreted economically as skewness and scale in Gibrat's (1931) proportionate growth, or inequality and scale of wage levels. (Salem and Mount 1974, 1115) [7] It is also the maximum entropy distribution of a random variable with fixed expected value and expected logarithmic value [8].

It is obvious that a stochastic differential equation represents the first difference of the variable whose cross-sectional distribution is consequently derived. The first difference of any process reveals its distributional

[7] There is another least three other popular drift-diffusion models with known analytical parameter transformations in the cross-section distribution, the Vasicek (1977) process gives a log-normal, and the Courtadon (1982) process an inverse Gamma distribution. A geometrical Brownian motion with jumps finally converges to a double-Pareto distribution of incomes.(Fischer 2018)

[8] See Scharfenaker (2022) on the conjecture of drift-diffusion and maximum entropy distributions of profit rates and income.

properties absent persistent differences (such as an increasing time trend between years or individual effects between people). It allows for an analysis of the dynamic commonalities, and the distribution of dynamic effects. At the same time, first differences emphasize random noise with cross-sectional zero mean (white noise, as in the OLS linear regression model). This makes it necessary to show that, when we transform the differential process back into levels, the effect is not equivalent to white noise, but reveals further dynamics.

For the CIR model, the mean-reverting equalization process and level-dependency of random diffusion fulfill that requirement. This is further illustrated by the fact that the CIR model reveals a wage level distribution close to the observed one without using distributional wage data as inputs.

$$dX = \theta(\mu - X)dt + \sigma\sqrt{X}dWt \tag{1}$$

$$X \sim Gamma(\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} X^{\alpha-1} e^{-\beta X} \tag{2}$$

Common measures of inequality for the Gamma distribution, such as the Gini and Pietra coefficients or Theil's entropy, are available in closed form. (Kleiber and Kotz 2003, 165) The Gini coefficient can be approximated in terms of DDM parameters (See Equation 3). (Fischer 2018, 11)

$$Gini(X) \approx \frac{\sigma}{\sqrt{\theta\pi}} \frac{1}{\mu} \tag{3}$$

4.2 Economic interpretation of CIR parameters

The Cox-Ingersoll-Ross drift-diffusion model simultaneously captures equalization and diffusion. It explicitly models normal employment conditions, the speed of equalization (mean reversion), the extent of diffusion relative to equalization, and scales the extent of diffusion by the wage level as an expression of differential gravitational centers in one common turbulent process.

Firm competition has investment and thereby labor demand accelerate with higher profit rates, which turbulently equalize. In the same fashion, limits to wage increases rise with regulating profit rates and cost differentials between regulating and subdominant capitals. This displacement from the mean, and more generally, from workers' current position on the wage curve, is captured by the stochastic Wiener process dW_t. The extent of diffusion is captured by σ and scaled by $\sqrt{X}$ to increase with the wage level. Furthermore, $\sqrt{X}$ acts as measuring differential centers of gravitation for different skill levels, as a Wiener process with higher standard deviation is shifted away from the common mean.

As workers follow above-average wages (for "their" level of skill or occupation), and as limits to wage increases move back across the mean, wage increases move back towards the mean. This is captured by the mean reversion component $\mu - X$ and quantified by θ. In order to capture industrial, occupational and individual participation in the turbulent process, we plug individual wage income X_t, normalized by the yearly national average, and their year-over-year growth $\Delta X_t = X_t - X_{t-1}$ in the estimation.

In the economic interpretation of the CIR DDM wage growth model, μ acts as the common center of the equalization process. This is a different interpretation from the statistical "long-run mean", among other reasons because "normal conditions of labor", rather than the arithmetic mean, regulate labor markets. We have argued that differential cost of re-skilling constitute different gravitational centers corresponding to different occupations. However, if we re-write the differential centers as deviations from some common center, then μ corresponds to some normal occupation of reference.

θ is often called the mean reversion speed (Fischer 2018, 4), and is a parameter of how fast turbulent equalization takes place. Equalization is faster when labor conditions are fairly standardized (eg. through collective bargaining), when labor moves quickly towards more favorable employment or when competing capitalists keep up with the labor conditions of innovators.

σ is the common standard deviation of the diffusion process, which is then again scaled for the wage level by $\sqrt(X)$, to reflect differential compensation between occupations. It describes the strength of diffusion, ie. relative movement along the wage curve in both directions. We would expect higher values of σ in periods of fast technological change, where new firms try to attract a lot of workers, and in industries with less labor protection. Keep in mind that σ is Gini-increasing (see Equation 3).

4.3 Bayesian estimation

$$\begin{aligned} X_{t+1} - X_t &= (1-\rho)\mu + (\rho - 1)X_t + \sigma\sqrt{X_t}W_t \\ \rho &\approx exp(-\theta) \end{aligned} \tag{4}$$

We rewrite the Cox-Ingersoll-Ross model in discrete form ($dX \equiv X_{t+1} - X_t$) and transform it into an auto-regressive time series estimation, as in Equation 4. Note that the process has convergence only for $0 < \rho < 1$.

One needs to explicitly model the Wiener process (and scale it by $\sqrt{X}$) in order to directly estimate σ. We use a Bayesian framework (Equation 5) to distinguish the Wiener process from regression uncertainty (the normally distributed error term in a frequentist AR(1) setup). We use individual wages X_t, wage growth $dX_{t+1} = X_{t+1} - X_t$ and lagged wage levels X_t as inputs. It is equivalent to a state space framework without a Kalman gain, which we refrain from due to the short length (two consecutive periods per individual observation). We furthermore normalize $x_t = X_t/\bar{X}_t$.

$$\begin{aligned} du_{i,t} &= (1-\rho)\mu + (\rho - 1)x_{i,t} + \sigma\sqrt{x_{i,t}}w_{i,t} \\ dx_{t+1} &\sim N(du_{i,t}, s) \\ w &\sim N(0,1) \end{aligned} \tag{5}$$

In state space terms, du is the state vector, dx the output vector, $(1-\rho)\mu + (rho - 1)x_t$ the state matrix, s represents the feed-forward matrix and we model an unit output matrix of diagonal one-entries. We furthermore introduce informative priors, where $Q_{0.5}(x_t)$ denotes the median normalized observation. We transform the CIR parameters into Gamma parameters within the model (without updating from the cross-sectional distribution however, as to estimate explanatory power from the drift diffusion approach alone), they have prior distributions too. We chose a Beta prior on ρ to restrict it to the space $[0,1]$ (necessary for a finite process), Cauchy priors for σ and s to allow for outliers, and exponential priors for the Gamma parameters α and β. For the estimation, we rely on a Hamiltonian Monte Carlo No U-turn sampler (HMC NUTS) in the software package STAN by Team (2020).

$$\begin{aligned} \rho &\sim Beta(2,5) \\ \mu &\sim Normal(Q_{0.5}(x), 0.5) \\ \sigma &\sim Half - Cauchy(0,1) \\ s &\sim Half - Cauchy(0,0.1) \\ \alpha &\sim Exponential(0.015) \\ \beta &\sim Exponential(0.015) \end{aligned}$$

The model estimates the distribution of wages without using distributional or aggregate wage data as inputs. Wage levels only enter the estimation equations in discretization and as a scaling factor $\sqrt{x}$ for the diffusion process standard deviation. The corresponding cross-sectional distribution comes only from individual year-over-year worker wage changes, and no curve fitting to observed level inequality. Goodness-of-fit statistics of the model therefore reflect how much of wage inequality can be explained by the turbulent dynamics of wage increases (when compared to some non-informative baseline scenario).

This novel estimation technique is parsimonious but can be easily adapted and extended (as the rich literature on drift-diffusion models show). The clear divide between wage increase data in the estimation, and comparison to wage levels after parameter transformation also serve as a proof-of-concept: Modeling the equalizing and turbulent dynamics of wage increases in a unified setup give a statistical and economic explanation for the stable form of the wage curve. Due to the intuitive economic interpretation of convergence and diffusion parameters in competition, adaptation to more complex models or industrial idiosyncrasies is simple.

4.4 Hierarchical modeling

Hierarchical models are an intuitive addition in Bayesian econometrics. They include heterogeneous parameters between groups, and order those in a hierarchy, eg. regional coefficients ordered by county, state and nation. This can be implemented by including the higher-order entity into the prior distribution of the lower-level coefficient, such that $p(\theta_i) \sim F(\theta) \forall i$. (Gelman et al. 2013, 101)

In our model this allows us to vary mean reversion θ and diffusion standard deviation σ either with the economic situation (by a time index) or industrial idiosyncrasies (with an industrial index), see Equation 6. We do not index μ because we model a common center of equalization. All combinations of yearly, industrial or no index $j \in J$ gives nine models. The hierarchical hyperprior on the indexed parameters, allows us to still analytically derive the cross-sectional distribution parameters α and β.

$$
\begin{aligned}
du_{i,t} &= (1-\rho_J)\mu + (\rho_J - 1)x_{i,t} + \sigma_J \sqrt{x_{i,t}} w_{i,t} \qquad (6) \\
dx_{t+1} &\sim N(du_{i,t}, s) \\
w &\sim N(0,1)
\end{aligned}
$$

$$
\begin{aligned}
& \qquad (7) \\
\rho_J &\sim Normal(\rho, 0.1) \\
\rho &\sim Beta(2,5) \\
\mu &\sim Normal(Q_{0.5}(x), 0.5) \\
\sigma_J &\sim Normal(\sigma, 0.1) \\
\sigma &\sim Half-Cauchy(0,1) \\
s &\sim Half-Cauchy(0,0.1) \\
\alpha &\sim Exponential(0.015) \\
\beta &\sim Exponential(0.015)
\end{aligned}
$$

5 Data

The IPUMS current population survey's annual social and economic supplement (CPS-ASEC) provides detailed demographic and employment structural information, as well as monthly wage income, on the individual level. Due to the structure of the annual socio-economic supplement, we can retrieve wage income from the same month in two subsequent years, as well as work hours, for each worker. We construct a worker-level sample spanning from 1998 – 2018 by calculating hourly wages and year-over-year increases of hourly wages. This is the basis for the individual level drift-diffusion model we estimate in section 4.1.

The observations can be aggregated to the industrial level and combined with industry-structural indicators such as regulating profit rates, capital-labor ratios and share of labor cost in total cost from BEA industry accounts (Tables "Components of Value Added by Industry", Table 6.4D and 6.5D on employment, Table 6.7D on self-employment) for the Vaona (2011) test of turbulent equalization.

In data selection, we follow the arguments in Shaikh (2008, 191) and Mokre and Rehm (2020, 924). We do not consider observations from the government or non-profit sectors due to interest in the interaction of profit-maximizing firms and wages. We furthermore exclude financial, insurance and real estate sectors due to difficulties in estimating inventory, which is in turn crucial for evaluating the turbulent behavior of profit rates. As we argue that turbulent equalization of profit rates (Shaikh 2008) and wages (Mokre and Rehm 2020) link the two processes and induce an drift-diffusion process, we further restrict the sample to industries participating in turbulent equalization in both categories. The three steps reduce the sample to 26 out of 53 NACE 2-digit industries, which however cover 81 % of employees in the original sample. (Mokre 2021, 14)

IPUMS uses top coding for income, ie. income above some fixed threshold value would be either set to said value (1962 – 1995), later set to a demographic group average (1996 – 2010) and since 2011 swapped with incomes from a similar observation rank (IPUMS 2020; Mokre 2021, 10). All three distortion measures decrease observed inequality (Shaikh, Papanikolaou, and Wiener 2014, 55).

6 Results

6.1 Goodness of Fit and Explanatory Power

Soofi's index of information distinguishability compares two distributions (eg. an observed distribution p and predicted distribution q), as in Equation 8. (Soofi, Ebrahimi, and Habibullah 1995) It represents the share of observed information not predicted (Wiener 2020) and doubles as a measure of goodness of fit as well as explanatory power. Soofi's IID uses the Kullback-Leibler divergence D_{KL} as in Equation 9, the weighted sum deviations between prediction and observation in $k \in K$ states. We choose the ratio of individual income to yearly weighted average income, in percentiles, as discrete states. However, the results are robust to smaller and larger bins. Note that the KLD never turns negative for proper distributions due to Gibb's inequality. (Falk 1970)

$$IID = 1 - e^{-D_{KL}(p,q)} \tag{8}$$

$$D_{KL}(p,q) = \sum_k p_k log\left(\frac{p_k}{q_k}\right) \tag{9}$$

For comparison, we also calculate root mean squared distances between p and q for the same percentile bins, where $RMSD = \sqrt{\frac{(p_k - q_k)^2}{K}}$.

6.2 Workhorse Model

The workhorse model in 5 estimates a drift diffusion model with posterior mean estimates $\theta = 1.29$, $\mu = 0.77$ and $\sigma = 0.92$ (see Table 1).

The $\hat{R}$ diagnostic of algorithmic convergence lies below 1.01 for all three parameters, (the STAN development team suggests discarding results with $\hat{R}$ above 1.05 in the corresponding R documentation[9], while Dan Simpson claims the team usually finds a value above 1.01 alarming [10]) . Effective sample size per parameter far exceeds the rule-of-thumb of five times the number of chains recommended by the STAN development team. (Gelman et al. 2013, 284ff) We run the usual HMC diagnostics (divergent iterations, saturated tree depths and E-BMFI) on the fit and find no indications for pathological behavior. The posterior plot of the main parameters in Figure 1 indicate wide posteriors and fat tails in variance terms σ and s, but the posterior distributions are unimodal, and with little deviation between the mode and mean of the distribution.

[9] https://mc-stan.org/rstan/reference/Rhat.html accessed on June 10, 2022

[10] https://statmodeling.stat.columbia.edu/2019/03/19/maybe-its-time-to-let-the-old-ways-die-or-we-broke-r-hat-so-now-we-have-to-fix-it/ accessed on June 10, 2022

Table 1: Estimates for DDM parameters, workhorse model ("DDM1"). Posterior mean, MCMC standard error, 10th, 50th and 90th percentiles as well as effective sample size and $\hat{R}$ convergence statistic.

	Mean	MCMC SE	10 %	50 %	90 %	ESS	$\hat{R}$
θ	1.29	0.02	0.62	1.18	2.16	653.80	1.00
μ	0.78	0.02	0.27	0.73	1.37	364.14	1.00
σ	0.92	0.05	0.31	0.71	1.77	258.55	1.01
s	1.66	0.11	0.54	1.10	2.99	303.59	1.01

Note:
HMC estimation with RSTAN.

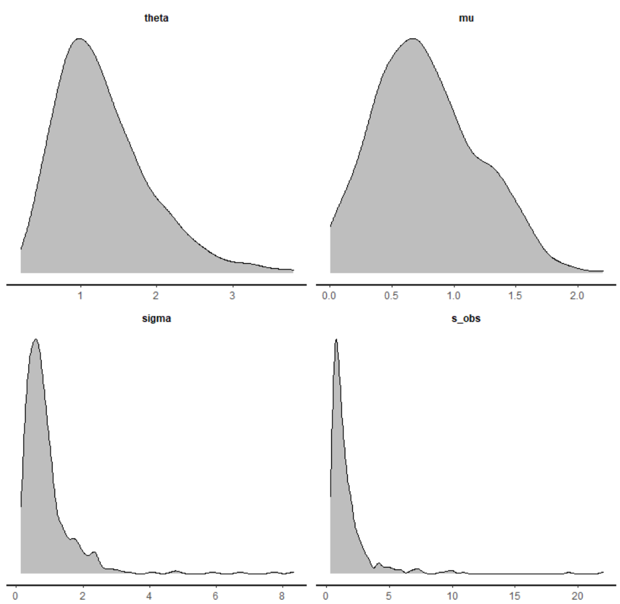

Figure 1: Posterior density of DDM parameters and regression error.

The Gamma parameters derived from these point estimates are $\alpha = \bar{\mu}\frac{2\bar{\theta}}{\bar{\sigma}^2} = 2.37$ and $\beta = \frac{2\bar{\theta}}{\bar{\sigma}^2} = 3.05$ where $\bar{x}$ denotes the posterior mean of a parameter x. The Soofi IID between a $Gamma(2.37, 3.05)$ predicted distribution and the observed wage curve is 0.1432 for the full distribution, which suggests the drift diffusion model explains more then 85 % of wage inequality. For the lower 99 % of wages the statistic drops to 0.0827. The coverage of the bottom 99 % might be the more important goodness-of-fit statistic for two reasons: One is the topcoding of wage income in the CPS, the second being the presence of Pareto distribution tails in wages. (Fischer 2018, 13; Courtadon 1982)

The fit is considerably worse for the direct posterior means of $\bar{\alpha}$ and $\bar{\beta}$ estimated within the algorithm, which are sensitive to outliers. The hierarchical (hyperprior) setup laid out in Equation 6 solves this problem. However this means that in the workhorse model we cannot evaluate the full posterior uncertainty over α and β, but only the underlying uncertainty in θ, μ and σ.

6.3 Hierarchical Model

In the hierarchical model (Equation 6), we have industrial or yearly heterogeneous parameters θ_J and/or σ_J drawn from a prior distribution around a common means θ and σ. This corresponds to either industrial idiosyncrasies in competition (for example, an industry-specific mean reversion could refer to more rigid wages due to traditionally strong unions) or the economic situation (eg. lesser labor mobility in a beginning recession). With regards to the statistical model, hyperpriors explicitly account for systematic outliers. This also means that statistical uncertainty does not “spill over”, if it occurs mostly in one closed group of observations, which improves algorithmic performance in Hamiltonian Monte Carlo settings.

There are nine possible combinations of yearly and industrial indexes (including the workhorse model with neither index). The model have similar goodness of fit statistics, with full-sample Soofi IIDs between 0.143 and 0.167 (see Table 2), and between 0.072 and 0.12 for the bottom 99 wage percentiles (See Table 3). Appendix Table 4 summarizes the DDM and Gamma distribution parameter estimates for the workhorse as well as the hierarchical models. We show the three best fits in figure 2. The worst fit model stands at a Soofi IID of 0.28, ie. 72 % of the variation explained. The ranking remains the same between the full sample and the bottom 99 %, and as expected, Soofi IID and RMSD give the same ranking of models.

In subsection 4.3 we showed that our Bayesian estimation of drift diffusion processes does not rely on the cross-sectional distribution of wages at all. It only takes into account wage levels to scale the impact of the diffusion process on wage growth (as is customary for the Cox, Ingersoll, and Ross (1985) model and consistent with the empirical findings in Mokre (2021)). Maybe even more importantly, the estimation does not take into account the macro-distribution of wages as we make no use of aggregate data (eg. percentile histograms, wage sums, or industrial distribution of wages). Thus, the high explanatory power of both the workhorse and the hierarchical models illustrate just how important wage growth is for wage level distribution. It lends support to the hypothesis that wage inequality is largely driven by turbulent equalization in firm and worker competition.

The workhorse model performs best, while the introduction of time-specific σ or industrial θ produce almost the same parameters for the gamma distribution as well as very similar goodness-of-fit statistics. The best-fit models are the ones with the highest α estimates, ie. the lowest inequality. As mentioned above, in the hierarchical model, posterior mean estimates $\bar{\alpha}$ and $\bar{\beta}$ for the Gamma parameters provide a good fit. The advantage of the hierarchical model is being able to evaluate the full posterior of the Gamma parameters and algorithmic stability, rather than improved parameter estimates.

Table 4: Estimates for DDM parameters. Posterior mean, MCMC standard error, 10th, 50th and 90th percentiles as well as effective sample size and $\hat{R}$ convergence statistic.

	Mean	MCMC SE	10 %	50 %	90 %	ESS	$\hat{R}$
DDM1 Workhorse Model							

Table 4: Estimates for DDM parameters. Posterior mean, MCMC standard error, 10th, 50th and 90th percentiles as well as effective sample size and $\hat{R}$ convergence statistic. *(continued)*

	Mean	MCMC SE	10 %	50 %	90 %	ESS	$\hat{R}$
θ	1.29	0.02	0.62	1.18	2.16	653.80	1.00
μ	0.78	0.02	0.27	0.73	1.37	364.14	1.00
σ	0.92	0.05	0.31	0.71	1.77	258.55	1.01
s	1.66	0.11	0.54	1.10	2.99	303.59	1.01
α	2.38						
β	3.05						
DDM2, Industry-Convergence, General Diffusion							
θ	1.01	0.04	0.55	1.00	1.46	87.45	1.01
μ	0.76	0.01	0.38	0.78	1.10	527.50	1.00
σ	0.89	0.03	0.30	0.70	1.64	429.79	1.00
s	1.62	0.10	0.55	1.09	3.01	326.29	1.00
α	1.96						
β	2.57						
DDM3 General Convergence, Industry-Diffusion							
θ	1.43	0.03	0.71	1.30	2.26	462.73	1.00
μ	0.78	0.01	0.41	0.78	1.14	545.65	1.00
σ	1.14	0.14	0.61	0.95	2.03	19.40	1.03
s	1.94	0.21	0.61	1.22	3.34	333.86	1.01
α	1.72						
β	2.21						
DDM4 Time-Convergence, General Diffusion							
θ	0.93	0.05	0.49	0.91	1.46	63.05	1.01
μ	0.76	0.01	0.42	0.77	1.06	635.14	1.00
σ	0.87	0.04	0.29	0.67	1.62	482.31	1.00
s	1.61	0.12	0.54	1.04	3.00	263.55	1.00
α	1.88						
β	2.48						
DDM5 General Convergence, Time-Diffusion							
θ	1.40	0.03	0.67	1.29	2.30	501.01	1.00
μ	0.77	0.01	0.37	0.74	1.20	560.23	1.00
σ	1.04	0.06	0.60	0.96	1.57	42.72	1.01
s	1.57	0.10	0.63	1.13	2.68	299.99	1.01
α	1.99						
β	2.60						
DDM6 Industry-Convergence, Industry-Diffusion							
θ	1.08	0.07	0.62	1.07	1.60	25.57	1.09
μ	0.77	0.01	0.39	0.76	1.15	690.75	1.00
σ	1.16	0.14	0.60	1.00	1.96	18.25	1.01
s	1.72	0.14	0.64	1.30	3.12	175.93	1.00
α	1.22						
β	1.59						
DDM7 Time-Convergence, Time-Diffusion							
θ	1.23	0.05	0.74	1.24	1.72	57.23	1.01
μ	0.78	0.01	0.40	0.78	1.17	845.96	1.00
σ	1.06	0.09	0.55	0.90	1.81	34.65	1.00
s	1.67	0.11	0.60	1.21	3.01	292.71	1.00
α	1.71						

Table 4: Estimates for DDM parameters. Posterior mean, MCMC standard error, 10th, 50th and 90th percentiles as well as effective sample size and $\hat{R}$ convergence statistic. *(continued)*

	Mean	MCMC SE	10 %	50 %	90 %	ESS	$\hat{R}$
β	2.20						
DDM8 Industry-Convergence, Time-Diffusion							
θ	1.04	0.04	0.54	1.07	1.50	67.99	1.00
μ	0.76	0.02	0.37	0.77	1.14	276.77	1.00
σ	1.25	0.12	0.61	1.06	2.26	29.48	1.06
s	1.91	0.16	0.63	1.36	3.67	164.38	1.00
α	1.02						
β	1.34						
DDM9 Time-Convergence, Industry-Diffusion							
θ	1.05	0.04	0.63	1.02	1.51	96.07	1.00
μ	0.78	0.02	0.40	0.79	1.13	305.30	1.00
σ	1.14	0.11	0.59	0.95	1.95	29.43	1.03
s	1.89	0.16	0.61	1.25	3.52	175.93	1.00
α	1.27						
β	1.63						

Note:
HMC estimation with RSTAN on 4 chains and 500 iterations after 1000 warmup iterations.

7 Conclusion

The competition of capitals and workers are intimately linked, their dynamics provide the framework in which wages are set and workers are hired or fired. In processes of turbulent equalization, we observe a slow, fundamental tendency of equalizing a center of gravitation, as well as systematic turbulence around it. These simultaneous dynamics of equalization and dis-equalization are the core of classical political economics models of profitability as well as wage inequality.

In this paper, we discussed Smith (1999)'s and Marx' (1990, 1993b, 1993a) theories of wage levels and wage differences respectively. In contrast to neo-classical theories, in their classical political economics models the wage level and wage differentials are results of the same process, a *constant equalization of ever-renewed inequalities*. This simultaneous equalization and dis-equalization follows the economic logic of capital accumulations and political struggles in wage bargaining.

We combine Botwinick (1993)'s model of bargaining under real competition with Shaikh (2020)'s model of turbulent wage dynamics. Botwinick (1993) shows that the limits to wage increases are subject to between-industry and within-industry competition. They can furthermore be decomposed into turbulently equalizing (regulating profit rates and within-industry cost differentials) and persistently different factors (capital intensity and share of labor cost in total cost). Shaikh (2020) propose that a drift-diffusion models capture the simultaneous equalization and dis-equalization of wages between industries. They furthermore note that a Cox, Ingersoll, and Ross (1985) model of growth rates asymptotically implies a cross-sectional Gamma distribution of levels. Our model presents a unified theory of turbulent wage inequality under real competition limits to wage bargaining as well as an estimation strategy based in wage increases.

Finally, e develop a novel Bayesian estimation method for the structural drift-diffusion model. The cross-sectional distribution of wage levels can be derived from the drift-diffusion parameters through direct parameter transformation, without using aggregate nor distributional wage data in the estimation procedure. This is a logical extension of the models in Adrian Drăgulescu and Yakovenko (2001), Shaikh, Papanikolaou, and Wiener (2014) and Shaikh (2020), which fit exponential and Gamma distributions to gross income. The

Table 2: Soofi IID and RMSD goodness-of-fit comparison of workhorse and hierarchical drift-diffusion models.

Model	α	β	Mean $\hat{R}$	Soofi IID	Rank Soofi	RMSD	Rank RMSD
θ, σ	2.38	3.05	1.00	0.14	1	0.0020	1
θ, σ_T	1.99	2.60	1.00	0.16	2	0.0021	2
θ_I, σ	1.96	2.57	1.00	0.16	3	0.0022	3
θ_T, σ	1.88	2.48	1.00	0.17	4	0.0022	4
θ, σ_I	1.72	2.21	1.01	0.17	5	0.0023	5
θ_T, σ_T	1.71	2.20	1.00	0.17	6	0.0023	6
θ_T, σ_I	1.27	1.63	1.01	0.22	7	0.0026	7
θ_I, σ_I	1.22	1.59	1.03	0.24	8	0.0026	8
θ_I, σ_T	1.02	1.34	1.02	0.28	9	0.0028	9

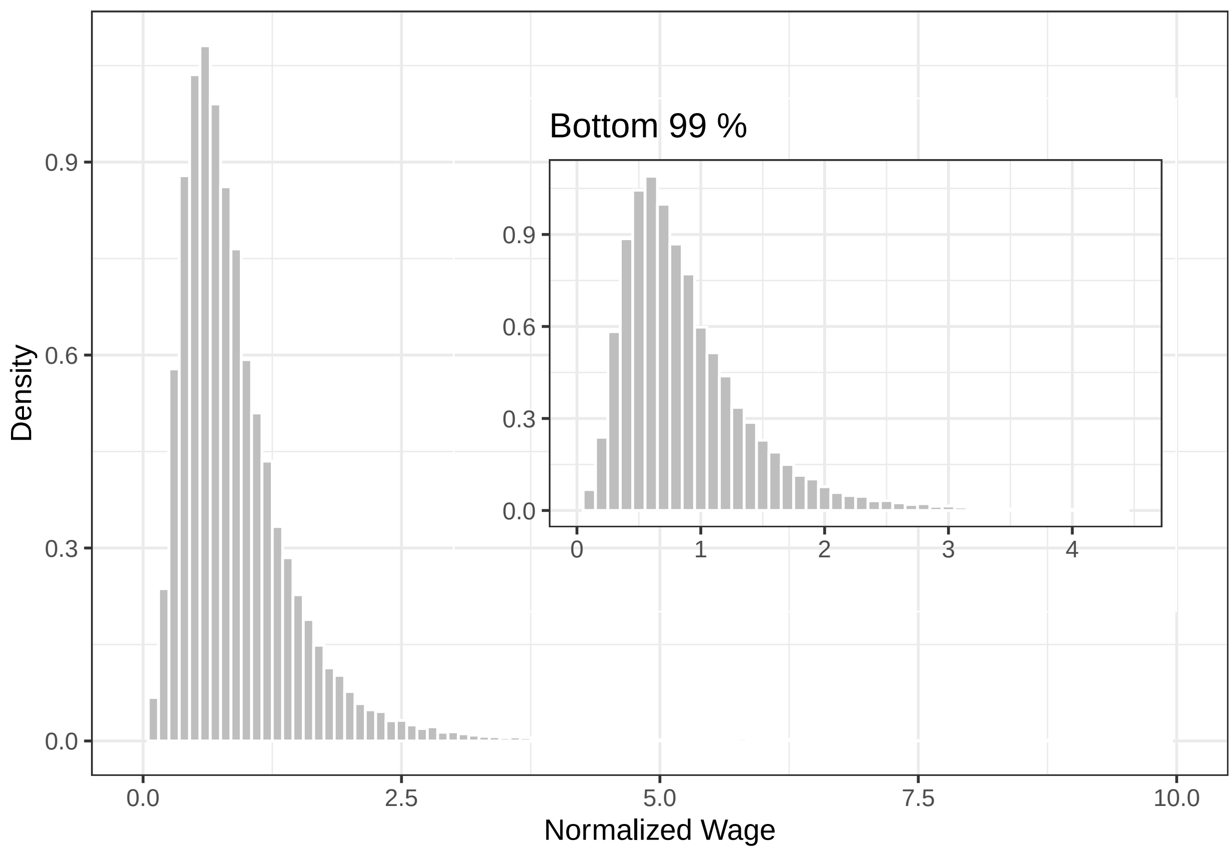

Figure 2: Histogram of normalized wages and best-fit distributions, full sample and bottom 99 pecentiles (inlay plot)

Table 3: Soofi IID and RMSD goodness-of-fit comparison of workhorse and hierarchical drift-diffusion models, for the bottom 99 % of wages

Model	α	β	Mean $\hat{R}$	Soofi IID	Rank Soofi	RMSD	Rank RMSD
θ, σ	2.38	3.05	1.00	0.08	1	0.0027	1
θ, σ_T	1.99	2.60	1.00	0.11	2	0.0029	2
θ_I, σ	1.96	2.57	1.00	0.12	3	0.0029	3
θ_T, σ	1.88	2.48	1.00	0.13	4	0.0030	4
θ, σ_I	1.72	2.21	1.01	0.14	5	0.0031	5
θ_T, σ_T	1.71	2.20	1.00	0.14	6	0.0031	6
θ_T, σ_I	1.27	1.63	1.01	0.21	7	0.0035	7
θ_I, σ_I	1.22	1.59	1.03	0.22	8	0.0036	8
θ_I, σ_T	1.02	1.34	1.02	0.27	9	0.0039	9

Gamma distribution we derive from the estimation has a Soofi, Ebrahimi, and Habibullah (1995) index of information distinguishability (IID) of 0.14 from the empirical distribution, which suggests that the drift diffusion model explains 86 % of overall wage inequality. By the same measure, over 92 % of the inequality in the bottom 99 percentiles can be explained.

This paper has two main contributions, in combining the main arguments in the real competition literature on wage inequality and as an empirical proof-of-concept. Drift diffusion models of wage inequality, and the link between profit rate and wage dynamcis, are coherent with the classical political economists' approaches to the matter. They take up Smith's (1999, 163) wage differentials between old and new trades as well as Marx' (1993a, 651) competition among workers and turbulent dynamics in the establishment of the general profit rate. These models also relate to the literature of stochastic growth (Gibrat 1931; Adrian Drăgulescu and Yakovenko 2001; Piketty and Saez 2006) and econophysics (Gabaix 2009; Fischer 2018).

As we have shown the theoretical coherence and empirical precision of the procedure, a number of intuitive extensions are possible (and necessary). Distinction between different qualities of labor, and correspondingly differential mechanisms of recruitment from unemployment or competing firms can be investigated separately. One can also incorporate job differences in a drift-diffusion setting beyond the square-root-of-wages term in the diffusion process, for example by introducing a Laplacian jump motion for job loss (Hainaut 2017). Furthermore, the model in this paper is a model of wage differentials rather than of wage levels. Another natural extension would be an auto-regressive process in the center of gravitation μ with the Gamma parameter of scale β as a scaling factor to macroeconomic variables that have an impact on inequality, such as the level of unemployment. (Watson 2002) Finally, the relationship between social oppression and wage inequality, eg. gender wage and racial wage gaps can be investigated explicitly within the theoretical and empirical model. We can model the interaction of women and racially oppressed groups being placed in below-average paying industries, as well as individual "shifts" along the wage distribution, as outlined in Shaikh, Papanikolaou, and Wiener (2014).

Literature

Acemoglu, Daron. 2002. "Technical Change, Inequality, and the Labor Market." *Journal of Economic Literature* 40 (1): 7–72. https://doi.org/10.1257/jel.40.1.7.

Ahlquist, John S. 2017. "Labor Unions, Political Representation, and Economic Inequality." *Annual Review of Political Science* 20 (1): 409–32. https://doi.org/10.1146/annurev-polisci-051215-023225.

Amoroso, Luigi. 1925. "Ricerche Intorno Alla Curva Dei Redditi." *Annali Di Matematica Pura Ed Applicata* 2: 123–59. https://doi.org/10.1007/BF02409935.

Ashenfelter, Orley, David Card, Henry Farber, and Michael Ransom. 2021. "Monopsony in the Labor Market: New Empirical Results and New Public Policies." w29522. Cambridge, MA: National Bureau of Economic Research. https://doi.org/10.3386/w29522.

Banerjee, Anand, Victor M. Yakovenko, and T. Di Matteo. 2006. "A Study of the Personal Income Distribution in Australia." *Physica A: Statistical Mechanics and Its Applications* 370 (1): 54–59. https://doi.org/10.1016/j.physa.2006.04.023.

Botwinick, Howard. 1993. *Persistent Inequalities: Wage Disparity Under Capitalist Competition.* Princeton, N.J: Princeton University Press.

———. 2018. *Persistent Inequalities: Wage Disparity Under Capitalist Competition.* Historical Materialism Book Series. Leiden ; Boston: Brill.

Bowles, Samuel. 2006. *Microeconomics: Behavior, Institutions, and Evolution.* The Roundtable Series in Behavioral Economics. Princeton, NJ: Princeton Univ. Press [u.a.].

Braverman, Harry. 1974. "Labor and Monopoly Capital." *New York: Monthly Review.*

Card, David. 1992. "The Effect of Unions on the Distribution of Wages: Redistribution or Relabelling?" w4195. Cambridge, MA: National Bureau of Economic Research. https://doi.org/10.3386/w4195.

Christensen, Björn. 2002. "Reservation Wages, Offered Wages, and Unemployment Duration: New Empirical Evidence." Working {{Paper}}. Kiel: Kiel Institute of World Economics (IfW).

Courtadon, Georges. 1982. "The Pricing of Options on Default-Free Bonds." *The Journal of Financial and Quantitative Analysis* 17 (1): 75. https://doi.org/10.2307/2330930.

Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross. 1985. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica* 53 (2): 363. https://doi.org/10.2307/1911241.

DiNardo, John, Nicole M. Fortin, and Thomas Lemieux. 1996. "Labor Market Institutions and the Distribution of Wages, 1973-1992: A Semiparametric Approach." *Econometrica* 64 (5): 1001. https://doi.org/10.2307/2171954.

Doeringer, Peter B., and Michael J. Piore. 1971. *Internal Labour Markets and Manpower Analysis.* Lexington, MA: Heath Lexington Books.

Drăgulescu, Adrian, and Victor M. Yakovenko. 2001. "Exponential and Power-Law Probability Distributions of Wealth and Income in the United Kingdom and the United States." *Physica A: Statistical Mechanics and Its Applications* 299 (1-2): 213–21. https://doi.org/10.1016/S0378-4371(01)00298-9.

Drăgulescu, A., and V. M. Yakovenko. 2001. "Evidence for the Exponential Distribution of Income in the USA." *The European Physical Journal B* 20 (4): 585–89. https://doi.org/10.1007/PL00011112.

Falk, H. 1970. "Inequalities of J. W. Gibbs." *American Journal of Physics* 38 (7): 858–69. https://doi.org/10.1119/1.1976484.

Fischer, Thomas. 2018. "Modeling Inequality and Mobility with Stochastic Processes," August.

Fokker, Adriaan Daniel. 1914. "Die Mittlere Energie Rotierender Elektrischer Dipole Im Strahlungsfeld." *Annalen Der Physik* 43 (4): 810–20.

Freeman, Richard B., and Lawrence F. Katz, eds. 1995. *Differences and Changes in Wage Structures.* NBER Comparative Labor Markets Series. Chicago: University of Chicago Press.

Gabaix, Xavier. 2009. "Power Laws in Economics and Finance." *Annual Review of Economics* 1 (1): 255–94.

Galbraith, James K. 1998. *Created Unequal: The Crisis in American Pay.* A Twentieth Century Fund Book. New York: Free Press.

Gelman, A, J B Carlin, H S Stern, D B Dunson, A Vehtari, D B Rubin, R Christensen, et al. 2013. "Bayesian Data Analysis, Third Edition," 656.

Gibrat, Robert. 1931. *Les Inégalités Économiques.* Paris: Recueil Sirey.

Hainaut, Donatien. 2017. "Continuous Mixed-Laplace Jump Diffusion Models for Stocks and Commodities." *Quantitative Finance and Economics* 1 (2): 145–73. https://doi.org/10.3934/QFE.2017.2.145.

IPUMS. 2020. "IPUMS CPS." https://cps.ipums.org/cps/topcodes_tables.shtml#note.

Kleiber, Christian, and Samuel Kotz. 2003. *Statistical Size Distributions in Economics and Actuarial Sciences.* Wiley Series in Probability and Statistics. Hoboken, NJ: Wiley.

Kolmogoroff, A. 1931. "Über die analytischen Methoden in der Wahrscheinlichkeitsrechnung." *Mathematische Annalen* 104 (1): 415–58. https://doi.org/10.1007/BF01457949.

Krueger, A. B. 1993. "How Computers Have Changed the Wage Structure: Evidence from Microdata, 1984-1989." *The Quarterly Journal of Economics* 108 (1): 33–60. https://doi.org/10.2307/2118494.

Lapides, Kenneth. 1998. *Marx's Wage Theory in Historical Perspective: Its Origins, Development, and Interpretation.* Westport (Conn.) London: Praeger.

Lee, D. S. 1999. "Wage Inequality in the United States During the 1980s: Rising Dispersion or Falling Minimum Wage?" *The Quarterly Journal of Economics* 114 (3): 977–1023. https://doi.org/10.1162/

003355399556197.
Lemieux, Thomas. 2007. "The Changing Nature of Wage Inequality." *Journal of Population Economics* 21 (1): 21–48. https://doi.org/10.1007/s00148-007-0169-0.
March, Lucien. 1898. "Quelques Exemples de Distribution Des Salaires." *Journal de La Société Française de Statistique* 39: 193–206.
Marx, Karl. 1990. *Capital Volume 1. A Critique of Political Economy.* Vol. 3. London: Penguin Books.
———. 1993a. *Capital Volume 3. A Critique of Polical Economy.* London: Penguin Books Ltd.
———. 1993b. *Grundrisse.* London: Penguin Books.
McDonald, James B. 1984. "Some Generalized Functions for the Size Distribution of Income." *Econometrica* 52: 647–63.
Mokre, Patrick. 2021. "The Quantile Impacts of Real Competition on Industrial Wage Inequality in the United States, 1998-2018." In *ECINEQ 2021*, 41. London.
Mokre, Patrick, and Miriam Rehm. 2020. "Inter-Industry Wage Inequality: Persistent Differences and Turbulent Equalisation." *Cambridge Journal of Economics* 44 (4): 919–42. https://doi.org/10.1093/cje/bez064.
Pareto, Vilfredo. 1897. *Cours d'Economic Politique.* Lausanne.
Piketty, Thomas, and Emmanuel Saez. 2006. "The Evolution of Top Incomes: A Historical and International Perspective." *American Economic Review* 96 (2): 200–205. https://doi.org/10.1257/000282806777212116.
Planck, Max. 1917. "Über Einen Satz Der Statistischen Dynamik Und Seine Erweiterung in Der Quantentheorie." *Sitzungsberichte Der Preussischen Akademie Der Wissenschaften Zu Berlin* 24: 324–41.
Ragab, Amr. 2014. "Three Essays on the Incomes of the Vast Majority." PhD thesis, New York: The New School.
Salem, A. B. Z., and T. D. Mount. 1974. "A Convenient Descriptive Model of Income Distribution: The Gamma Density." *Econometrica* 42 (6): 1115. https://doi.org/10.2307/1914221.
Scharfenaker, Ellis. 2022. "STATISTICAL EQUILIBRIUM METHODS IN ANALYTICAL POLITICAL ECONOMY." *Journal of Economic Surveys* 36 (2): 276–309. https://doi.org/10.1111/joes.12403.
Schumpeter, Joseph A., Elizabeth Schumpeter, and Mark Perlman. 1997. *History of Economic Analysis.* Edited by Elizabeth B. Schumpeter. Reprint. Economics, History of Ideas. London: Routledge.
Shaikh, Anwar. 1980. "Marxian Competition Versus Perfect Competition: Further Comments on the so-Called Choice of Technique." *Cambridge Journal of Economics* 4 (1): 75–83.
———. 2008. "Competition and Industrial Rates of Return." In *Issues in Finance and Industry: Essays in Honour of Ajit Singh*, 167–74.
———. 2016. *Capitalism: Competition, Conflict, Crises.* Oxford ; New York, NY: Oxford University Press.
———. 2020. "The Econ in Econophysics." *The European Physical Journal Special Topics* 229 (9): 1675–84. https://doi.org/10.1140/epjst/e2020-900204-5.
Shaikh, Anwar, and Kyle Glenn. 2018. "The Classical Treatment of Skilled Labor." Working {{Paper}} 11/2018. New York: Department of Economics. The New School for Social Research.
Shaikh, Anwar, and Juan Esteban Jacobo. 2020. "Economic Arbitrage and the Econophysics of Income Inequality." *Review of Behavioral Economics* 7 (4): 299–315. https://doi.org/10.1561/105.00000129.
Shaikh, Anwar, Nikolaos Papanikolaou, and Noe Wiener. 2014. "Race, Gender and the Econophysics of Income Distribution in the USA." *Physica A: Statistical Mechanics and Its Applications* 415 (December): 54–60. https://doi.org/10.1016/j.physa.2014.07.043.
Smith, Adam. 1999. *The Wealth of Nations, Books I-III, Edited by Andrew Skinner.* London: Penguin Classics.
Soofi, Ehsan S., Nader Ebrahimi, and Mohamed Habibullah. 1995. "Information Distinguishability with Application to Analysis of Failure Data." *Journal of the American Statistical Association* 90 (430): 657–68. https://doi.org/10.1080/01621459.1995.10476560.
Team, STAN Development. 2020. *Stan 2.23 User's Guide.*
Tescari, Stefania, and Andrea Vaona. 2014. "Regulating Rates of Return Do Gravitate in US Manufacturing!: Regulating Rates of Return Do Gravitate in US Manufacturing!" *Metroeconomica* 65 (3): 377–96. https://doi.org/10.1111/meca.12043.
Vaona, Andrea. 2011. "An Empirical Investigation into the Gravitation and Convergence of Industry Return Rates in OECD Countries." *International Review of Applied Economics* 25 (4): 465–502. https:

//doi.org/10.1080/02692171.2010.529429.
Vasicek, Oldrich. 1977. "An Equilibrium Characterization of the Term Structure." *Journal of Fincancial Economics* 5: 177–88.
Watson, Ian. 2002. "Wage Inequality and Underemployment: Australia in the 1990s." *Journal of Industrial Relations* 44 (1): 88–107. https://doi.org/10.1111/1472-9296.00038.
Weeks, John. 2001. "The Expansion of Capital and Uneven Development on a World Scale." *Capital & Class* 25 (2): 9–30. https://doi.org/10.1177/030981680107400102.
Wiener, Noé M. 2020. "Labor Market Segmentation and Immigrant Competition: A Quantal Response Statistical Equilibrium Analysis." *Entropy* 22 (7): 742. https://doi.org/10.3390/e22070742.
Yellen, Janet L. 1984. "Efficiency Wage Models of Unemployment." *The American Economic Review* 74 (2): 200–205. https://www.jstor.org/stable/1816355.